COMPUTER-GENERATED HOLOGRAMS IMPROVED BY A GLOBAL ITERATIVE CODING

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Computer-generated holograms improved by a global iterative coding

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Abstract. A global iterative coding method for computer-generated holograms (CGH) is introduced. The method is based on the iterative correction of a CGH using standard Lee coding. The correction, i.e., the difference between the desired and the obtained reconstruction, is coded and added to the current CGH. The coding and weighting factors are introduced to control the speed of convergence and the SNR. Advantages lie in low computing time and improvement of the object SNR, the neighborhood SNR, and the background SNR. Reducing the standard deviation of the phase of the reconstructed object also results. A slight improvement of the diffraction efficiency is also observed. The comparison is realized using the Lee interferogram method as a reference. This method is tested on binary and complex gray-level objects. Simulation results and optical reconstruction are also presented.

Subject terms: computer-generated holograms; global iterative coding; spatial light modulators; neural networks; pattern recognition.

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1 Introduction

Since the initial work of Brown and Lohmann and Lohmann and Paris in 1966 and 1967, computer-generated holography has been the subject of constant research and interest. Much initial work has been oriented toward spatial filtering operations. Recent technological achievements in the fields of real-time spatial light modulators and diffractive optics have opened new ways of implementing optical recognition systems and optical neural networks. On the other hand, this has increased the interest for coding higher quality computer-generated holograms (CGHs). Spatial light modulators (SLMs) used for the real-time display of computer generated holograms are generally of different types, such as liquid crystal television (LCTV), magneto-optic SLMs (MOSLMs), or ferroelectric liquid crystal SLMs (FLC-SLM). All of these devices have their advantages and disadvantages, the main problem being the relatively small number of degrees of freedom available for CGH coding.

CGH research has always been motivated by the need to improve the quality of the information to achieve the best reconstruction possible. The precision of a CGH recording can be improved by increasing the number of elementary cells, which yields CGHs of large dimensions. As a result, these CGHs cannot be displayed on currently available SLMs. The challenge is therefore to develop a coding that can achieve a good reconstruction quality while maintaining the required number of degrees of freedom at a minimum. Such a coding method should allow the use of commercially available SLMs with sizes as small as 128 × 128 pixels.

Much work has been done to improve CGH coding. All this work has brought about various improvements, such as interpolation techniques, overflow correction, and the interferogram approach. We have proposed a method based on diagonal coding to use the degrees of freedom of support efficiently. This method was based on a 2 × 2 supercell. The direct binary search has paved the way for a new computing method and for a new way of considering computer-generated holograms. This method has been used to generate spot arrays and has been applied to electro-optical hologram generation. With this method, a CGH does not correspond to a coding formula but rather to an object computing based on an optimization criterion. This method yields good results because the reconstruction quality is mainly controlled by the restriction imposed on the optimization process. The main drawback of this method, however, is its computational in-
tensity, which requires long computation times, and its high residual noise level when the optimization is not performed.

The purpose of this paper is to propose a global iterative method to increase the quality of the reconstructed CGH while maintaining a good computational speed. This method also significantly reduces the noise in the neighborhood of the optimization region and slightly improves the diffraction efficiency. The method is demonstrated for binary computer-generated holograms and is extended to gray-level CGHs for future implementation on an LCTV modulator. An optical reconstruction is also performed and compared to that obtained by CGH simulation. The basic concept is to use the Lee method as a tool to code the hologram and to modify it by taking into account the error observed on the reconstruction. Because all the information of the reconstruction is used at each iteration, this method is considered global.

2 Computer-Generated Hologram

Hologram computation consists in generating a binary mask so that the propagated wave front will have the required form. For spatial filtering, the mask is considered to be in the frequency domain, whereas its Fourier transform is considered to be in the spatial domain. The purpose is then to record a CGH such that the Fourier transform will reveal the required object or wave front. In optics, the two domains are related by the continuous Fourier transform

\[ \mathcal{F}(h(x,y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x,y) e^{-j2\pi(ux+vy)} \, dx \, dy \]  

In the discrete form, for a hologram of dimensions \( M \times N \), with one cell representing one point of the Fourier transform, we refer to the Lee's interferogram method (1979). In this method, Lee identified the leading and the trailing edges of the fringe pattern with the following equation:

\[ \cos[2\pi u/T + \Phi(u,v)] = \cos[\pi q(u,v)] \]  

where \( T \) holds for the period and \( \Phi \) for the phase. With that location, one can just clip the function according to a bias function \( \cos[2\pi u/T + \Phi(u,v)] \), as shown in Fig. 1. The Fourier expansion of this clipped function gives, in the first order, the exact representation of the recorded wave front. Lee proposed three methods for choosing the \( \pi q(u,v) \) parameter. The first one concerns coding the phase and amplitude by the means of the position and the height of the fringes. The third one is basically a superposition of two phase-only holograms. In the second method, the function \( \pi q(m,n) \) is simply the corresponding angle of the normalized amplitude of the wave front. This can be expressed as

\[ \pi q(m,n) = \arcsin[A(m,n)/A_{\text{max}}] \]  

The clipping procedure of the binary mask is then expressed by

\[ H(u,v) = \begin{cases} 1 & \text{if } \cos[2\pi u/T + \Phi(m,n)] - \cos[\pi q(m,n)] \geq 0 \\ 0 & \text{otherwise} \end{cases} \]

where \( T \) is the period of the carrier used for the recording and \( \Phi \) and \( A \) are, respectively, the phase and the module of the Fourier transform at the frequency coordinates \((m,n)\). One advantage of this method is that it avoids the use of supercells, as needed in the Brown-Lohmann coding. These supercells usually require large coding space and reduce the number of degrees of freedom available on the hologram support. Lee's coding allows a one-to-one correspondence of the hologram encoded with the wave front to be recorded. However, Lee holograms usually show higher background noise than Brown-Lohmann CGHs. Although Lee coding was chosen for this one-to-one correspondence, which allows more flexibility, other codings could be used because the precision is increased by the iterative process.

3 Iterative Coding

To increase the performance of the CGH recording, some authors have proposed iterative methods to correct the recorded hologram. Let us introduce the desired object \( h_d(x,y) \) and the corresponding reconstructed region in the space domain \( h_r(x,y) \), which is a subregion of \( h(t,x,y) \) and which location depends on the choice of the carrier frequency. The iterative process must reduce the difference:

\[ \epsilon(x,y) = h_d(x,y) - h_r(x,y) = A(x,y) \exp[j\phi(x,y)] \]

where \( A(x,y) \) is the input object to be coded and \( t \) and \( t-1 \) correspond, respectively, to the current iteration and to the preceding iteration. The desired object \( h_d \) remains constant during the whole process and does not change with the iterative process.

The second method to correct the reconstructed object, one can add this difference to the input and expect to correct the output. Two methods are possible to globally correct the reconstruction. One is to act on the input object itself to compensate for the system degradation. This is mathematically expressed as

\[ h_s(x,y) = h_d(x,y) - \epsilon(x,y,t-1) \]

where \( h_s \) is the input object to be coded and \( t \) and \( t-1 \) correspond, respectively, to the current iteration and to the preceding iteration. The desired object \( h_d \) remains constant during the whole process and does not change with the iterative process.

The second method to correct the hologram is to modify the coding to directly change the reconstruction:

\[ H(u,v,t) = H_u(u,v,t) + H_t(u,v,t-1) \]

where \( H_u(u,v,t) \) is the hologram at iteration \( t \) and \( H_t \) is the hologram coding of the difference between the required and the obtained output. In this case, the modification is per-
formed on the hologram and only the difference of the pre-
ceding iteration is taken into account.

The first method is not practical because the difference 
added to the input object modifies the coded hologram glo-
ally. As a result, the correction to the coded object is no 
longer correct, because the whole hologram is changed. This 
gives rise to long converging times in addition to occasionally 
reducing the SNR measured on the reconstructed object.

The second method is the one we choose. In this case, the 
correction is added locally to the hologram, which does not 
modify the reconstruction on which the error has been com-
puted. The only problem encountered with this procedure is 
the fact that, because the hologram is a binary mask, adding 
a 1 over a pixel that is already 1 is impossible, so a pixel 
with an accumulated value higher than 1 will always be kept 
to a unit value. This nonlinear effect influences the conver-
gence. Because there is no further nonlinear operation after 
the correction of the hologram, the coding is more easily 
controlled.

The main process for computing the hologram is based 
on a modified version of Eq. (4):

\[
H(u,v) = \begin{cases} 
1 & \text{if } \cos(2\pi n/T + \Phi(m,n)) - \alpha \cos(\pi \Omega(m,n)) > 0 \\
0 & \text{otherwise}
\end{cases},
\]  

(8)

with

\[
\pi \Omega(m,n) = \arcsin[A(m,n)/A_{\text{max}}].
\]  

(9)

The correction process is given by

\[
H(u,v,t) = H(u,v,t-1) + H_e(u,v,t-1),
\]  

(10)

and the difference is computed

\[
e_t(x,y) = w_e h_e(x,y) - h_e(x,y) = w a \exp[j \phi_e(x,y)].
\]  

(11)

The coefficient \(\alpha\) in Eq. (8) is introduced to control the speed 
of convergence of the iterative process and is referred to as 
a coding factor. An \(\alpha\) value of 1 corresponds to the original 
Lee coding. When \(\alpha\) decreases, the probability that \(H(u,v)\) 
becomes 1 increases. Because the argument \(A(m,n)/A_{\text{max}}\) 
is normalized with a maximum value of 1, the value of \(\pi \Omega(m,n)\) 
is always positive. Decreasing \(\alpha\) in fact increases the number 
of fringes to be coded. The variable \(\alpha\) also has an effect on 
the background level, as we shall see. The next section treats 
more specifically the weighting factor \(w\) in Eq. (11), holding 
for \(w_0\) or \(w_v\).

4 Template

Three regions are considered for computing the correction, 
as illustrated in Fig. 2. The first region is the information 
to be coded and the immediate surroundings to be controlled.
This is identified by the white region and corresponds to 
the letter \(A\) with its immediate surroundings. It is referred to as 
the object. The second region is the portion delimited by the 
gray square and is called the neighborhood. The last region 
comprises everything else and is referred to as the back-
ground. When computing a correction for the next iteration, 
one can attribute a higher importance to a specific region.
We then define two weighting factors, the object coefficient 
\(w_o\) and the neighborhood coefficient \(w_n\). No factor is attrib-
uted to the background because no correction is applied to 
it. The main use of these factors is to enable correcting the 
object more than the neighborhood and to avoid the latter 
overloading the correction algorithm, with a resulting deg-
radation of the object, for the purpose of improving a region 
without useful information. Thus the correction \(e_t(x,y,t)\) 
is multiplied by the weighting factor corresponding to its lo-
cation in the template.

5 Experiments

The following experiments were computer simulations and 
optical reconstructions performed on different objects with 
128 × 128 pixels. Experiments were carried out on a Sparc 2 
workstation with an average computation time of 6.35 
seconds/iteration. Between 7 and 30 iterations were needed to obtain 
satisfactory results, depending mainly on the complexity of 
the object. This corresponds to a minimum and a maximum 
value of 44 and 189 s, respectively for a nonoptimized C 
written code.

The experiments compare the global iterative coding 
(GIC) to the Lee original coding. The comparison is made 
with the help of five parameters. The first is the SNR of the 
amplitude of the object measured on the object region pre-
viously defined. The second value is the SNR measured in 
the neighborhood of the object. The third value is the SNR 
in the background. The standard deviation of the phase of 
the reconstructed object is also measured along with the dif-
fract efficiency of the reconstruction. The latter is mea-
sured by integrating the energy of the object in the recon-
struction plane and by dividing the result by the total energy 
of the scene. The SNR on the object, the neighborhood, and 
the background are defined as

\[
\text{SNR}_{\text{OBJ}} = \frac{P_{\text{OBJ}}}{\sigma_{\text{OBJ}}^2},
\]  

(12)

\[
\text{SNR}_{\text{NEIG}} = \frac{P_{\text{NEIG}}}{P_{\text{OBJ}}},
\]  

(13)
Fig. 3 Behavior of the different SNR region of the reconstruction for a letter E with weighting factors of (a) 0.1, (b) 0.3, (c) 0.7, and (d) 1.0.

Table 1. Measured parameters of the Lee coding and the global iterative process (GIC) involving 20 iterations for different values of $\alpha$, $w_o = 0.75$ and $w_n = 0.25$.

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<thead>
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<tbody>
<tr>
<td>Original</td>
<td>2.72</td>
<td>3.10</td>
<td>4.32</td>
<td>2.68e-15</td>
<td>0.032</td>
</tr>
<tr>
<td>GIC $\alpha=1.0$</td>
<td>3.98</td>
<td>2.44</td>
<td>1.40</td>
<td>2.44e-01</td>
<td>0.011</td>
</tr>
<tr>
<td>GIC $\alpha=0.7$</td>
<td>5.93</td>
<td>4.70</td>
<td>5.27</td>
<td>1.14e-01</td>
<td>0.036</td>
</tr>
<tr>
<td>GIC $\alpha=0.3$</td>
<td>5.77</td>
<td>5.00</td>
<td>6.16</td>
<td>1.09e-01</td>
<td>0.041</td>
</tr>
<tr>
<td>GIC $\alpha=0.1$</td>
<td>4.24</td>
<td>4.63</td>
<td>7.71</td>
<td>1.80e-01</td>
<td>0.042</td>
</tr>
</tbody>
</table>

$\text{SNR}_{\text{BACK}} = \mu_{\text{BACK}} / \sigma_{\text{BACK}}$,  

where $\mu_{\text{BACK}}$ and $\sigma_{\text{BACK}}$ are the mean and the standard deviation of the amplitude, respectively.

Figure 3 and Table 1 present the iteration results for a letter E with four different coding factors, i.e., 1.0, 0.7, 0.3, and 0.1, for an object weighting of 0.75 and a neighborhood weighting of 0.25. Decreasing $\alpha$ reduces the number of iterations required to obtain the optimum point. Decreasing $\alpha$ also improves the SNR in the background region. Smaller values of $\alpha$ seem also to decrease the standard deviation of the phase while slightly increasing the diffraction efficiency. However, too small values of $\alpha$ decrease the SNR of the object and neighborhood. Considering this behavior, one chooses a lower coding factor (around 0.30) to obtain a faster convergence rate. This factor can be used when the object is relatively simple (uniform in phase and intensity). For complex objects (varying in phase and intensity), it is preferable to use a higher learning factor. The factor can, however, be predicted by considering the amount of correction applied to an object at each iteration and by maintaining this amount constant with a variation of the coding factor. With this procedure, one can automatically set the end of the iteration process by considering the variation of the different SNR values. When this variation becomes small or decreases, the iterative process can be ended. The weighting factors are needed to harmonize the optimum SNR of the object to the optimum SNR of the neighborhood. Once these factors have been adjusted for an image, they will be almost similar for other images of the same kind.

The behavior of the iteration process is presented in Fig. 4. The five iterations presented correspond to steps 1, 3, 7, 11, and 20 for a coding factor of 0.3. The original Lee coding is also presented for comparison. The holograms presented show that as the computing process progresses, the mask presents an increasing number of slits and transmits more light. This is sometimes not desirable because one prefers less light in the zeroth order so as not to saturate the detectors. According to the Babinet principle, the complementary mask is equivalent to the original one except for a phase shift of $\pi$ radians. This phase shift does not disturb the correlation function, which is the final purpose of the recording. This complementary mask has been computed for step 20 and is presented in Fig. 4(e). An interesting possibility would be to compute the CGH with the iterative process and to flip the result according to the Babinet principle. The iterative process could then again be performed on the modified CGH to improve the reconstruction again. However, a second iterative procedure is much harder to control because flipping the
Fig. 4 Hologram and the reconstruction for iterations (a) 1, (b) 3, (c) 7, (d) 11, (e) 20, and (f) for the Lee coding; GIC weighting factors are 0.75 (object) and 0.25 (neighborhood) with $\nu$ value of 0.3. Flipping of the hologram is performed in iteration 20 (e) (Table 1).
Fig. 4 Continued.
hologram introduces a discontinuity in the number of pixels changed in an iteration. The coding factor would then need to be changed. This aspect, however, is beyond the scope of this paper.

Figure 5 and Table 2 present the same kind of results for a letter A. In this case, a template, similar to that introduced previously, as been used to keep the edges of the letter well defined. Computed results show an improvement in all the measured values.

Table 2 Measured parameters for a letter A recorded with a template for 14 iterations and for the Lee coding. GIC parameters are $\alpha = 0.3$, $w_p = 0.80$, and $w_n = 0.20$.

<table>
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<tbody>
<tr>
<td>Original Lee</td>
<td>2.20</td>
<td>2.93</td>
<td>3.99</td>
<td>4.15 e-01</td>
<td>0.030</td>
</tr>
<tr>
<td>GIC</td>
<td>3.89</td>
<td>3.93</td>
<td>7.13</td>
<td>8.85 e-02</td>
<td>0.044</td>
</tr>
</tbody>
</table>

Figure 6 and Table 3 introduce the coding of a complex object with gray levels. Each pixel has a variable amplitude and a nonzero variable phase. The original letter E has a decreasing intensity from the top to the bottom. In this case, also, all the measured data improve compared to the standard Lee coding. This experiment also shows that the global iterative coding can be applied to arbitrary complex objects.

Table 3 Comparison of Lee coding and global iterative coding for a complex object with gray level. Number of iterations is 29. GIC parameters are $\alpha = 0.85$, $w_p = 0.95$, and $w_n = 0.05$.

<table>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Lee</td>
<td>5.23</td>
<td>3.16</td>
<td>3.17</td>
<td>2.44 e-01</td>
<td>0.026</td>
</tr>
<tr>
<td>GIC</td>
<td>6.27</td>
<td>3.12</td>
<td>5.03</td>
<td>1.64 e-01</td>
<td>0.032</td>
</tr>
</tbody>
</table>

Figure 6 and Table 3 introduce the coding of a complex object with gray levels. Each pixel has a variable amplitude and a nonzero variable phase. The original letter E has a decreasing intensity from the top to the bottom. In this case, also, all the measured data improve compared to the standard Lee coding. This experiment also shows that the global iterative coding can be applied to arbitrary complex objects.
The objects are letters E oriented at 90, 45, and 0 deg.

has always been a goal in fields such as pattern recognition and neural networks.

Fig. 7 Reconstruction of multiplexed objects for (a) Lee coding and (b) GIC. A template has been used in the GIC process (Table 4).

and neural networks.

Maximizing the use of the available degrees of freedom has always been a goal in fields such as pattern recognition and neural networks. It is possible to code more than one object on a CGH, thus allowing more processing in the same time with the help of the inherent parallelism of optics. Figure 7 shows an example of such multiplexing. In this recording, three images corresponding to three letters E with different orientation were coded. One can see that the global iterative coding gives a better-defined image with a higher light intensity, which corresponds to the predicted behavior of the coding. To obtain enough intensity, the hologram of the Lee coding was exposed four times longer than the GIC hologram. The uniformity of the letter is greater for the GIC coding than for the Lee coding.

Although mathematical convergence of the process is not presented, an intuitive explanation can be suggested as follows. Suppose a CGH can be recorded with an infinite number of gray levels. It would be possible to correct the reconstruction by successively adding the binary hologram of the correction to the original hologram. After 10 iterations, the holograms would possess 10 gray levels. This process is limited by the nonoverlapping space available in the reconstruction plane. The corresponding measured values are given in Table 4.

To verify the optical implementation of the presented results, a CGH was recorded on a simple slide by photographic means. Original Lee coding was compared to the E with weighting factors for the object and the neighborhood of 0.75 and 0.25, respectively, computed with a coding factor of 0.3. The holograms were illuminated with a collimated beam in the input plane, as shown in Fig. 8, but the reconstructed image obtained was recorded using a photographic film. The dc level was cut, but the intensity of the HeNe laser was kept constant for both tests. The results of Fig. 9 show that the global iterative coding gives a better-defined image with a higher light intensity, which corresponds to the predicted behavior of the coding. To obtain enough intensity, the hologram of the Lee coding was exposed four times longer than the GIC hologram. The uniformity of the letter is greater for the GIC coding than for the Lee coding.

The optical implementation of neural networks often takes advantage of low-cost LCTVs used as spatial light modulators. Such LCTV modulators may allow some gray-level coding. The number of gray levels, depending on the manufacturer, may be up to 8. The previously introduced iteration method can be used to correct the reconstruction. The correction is still presented by Eq. (8), however, there is no more binarization of the amplitude, there is only a clipping for values higher than 7. So, a correction is added at each iteration until pixels reach the value of 7. After the seventh iteration, the values of the pixels range between 0 and 7. For the following iteration, the correction is added to the hologram wherever the amplitude is lower than 7. If a correction increases a pixel value to 8, the pixel keeps its value of 7. The process is continued until a satisfying result is obtained. Figure 10 presents such a reconstruction with a coding factor of 0.2, with no correction for the background, and with 29 iterations. It is obvious that a small number of gray levels yields a good reconstruction even with a simple procedure, as can be seen by considering the data of Table 5.

Although mathematical convergence of the process is not presented, an intuitive explanation can be suggested as follows. Suppose a CGH can be recorded with an infinite number of gray levels. It would be possible to correct the reconstruction by successively adding the binary hologram of the correction to the original hologram. After 10 iterations, the holograms would possess 10 gray levels. This process is limited only by the coding used and by the nonlinear clipping procedure. Each iteration brings a correction that, although not complete, improves the reconstruction toward the desired object. The only change in the present method is that the coded CGH is binary and this introduces another clipping procedure for pixels with a value higher than 1. However, each iteration still tends to improve the reconstruction, yielding a better hologram.

### Table 4 Measured parameters for Lee coding and global iterative coding (eight iterations) for multiplexed objects. GIC parameters are \( \alpha = 0.25, \) \( w_e = 0.90, \) and \( w_w = 0.10. \)

<table>
<thead>
<tr>
<th>Multiple E</th>
<th>SNR Obj</th>
<th>SNR Neg.</th>
<th>SNR Backgr.</th>
<th>Stand. Dev.</th>
<th>Diffps.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Org. Lee E</td>
<td>1.58</td>
<td>1.63</td>
<td>1.87</td>
<td>1.09</td>
<td>0.013</td>
</tr>
<tr>
<td>GIC E1</td>
<td>3.50</td>
<td>3.22</td>
<td>3.17</td>
<td>1.15e-01</td>
<td>0.024</td>
</tr>
<tr>
<td>Org. Lee E2</td>
<td>1.60</td>
<td>1.56</td>
<td>1.82</td>
<td>1.12</td>
<td>0.012</td>
</tr>
<tr>
<td>GIC E2</td>
<td>3.22</td>
<td>3.18</td>
<td>3.16</td>
<td>1.43e-01</td>
<td>0.003</td>
</tr>
<tr>
<td>Org. Lee E3</td>
<td>1.66</td>
<td>1.86</td>
<td>2.13</td>
<td>6.82e-01</td>
<td>0.015</td>
</tr>
<tr>
<td>GIC E3</td>
<td>3.75</td>
<td>3.43</td>
<td>3.25</td>
<td>1.07e-01</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Fig. 7 Reconstruction of multiplexed objects for (a) Lee coding and (b) GIC. A template has been used in the GIC process (Table 4). The objects are letters E oriented at 90, 45, and 0 deg.
Fig. 8 Setup used in the experiment performed for the optical reconstruction of the CGHs.

Fig. 9 Optical results obtained with holograms of (a) Lee coding (hologram of Fig. 4(f) exposed 1/2 s) and (b) GIC (hologram of Fig. 4(e) exposed 1/2 s).
Fig. 10 Reconstruction of a hologram recorded with GIC for seven-gray-level (Table 5): (a) computed reconstruction and (b) optical reconstruction. Time of exposure: 1/10 s.

Table 5 Results for an E recorded with seven gray levels for 25 iterations. GIC parameters are $a = 0.20$, $W = 0.80$, and $w_n = 0.00$

<table>
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<tbody>
<tr>
<td>GIC</td>
<td>21.93</td>
<td>9.22</td>
<td>13.28</td>
<td>2.4 ± 0.02</td>
<td>0.060</td>
</tr>
</tbody>
</table>

6 Conclusion

A global iterative coding method has been presented. Although our coding is based on the Lee coding, other coding schemes could be used for the initial CGH. Corrections were applied directly to the CGH rather than to the coded object. Such a coding shows a good performance with respect to the measured parameters such as SNR, phase standard deviation, and diffraction efficiency. Another advantage of this method is the low computation time required. This time is related to the coding factor, which decreases the convergence time.

Aspects of this coding to be studied in the future concern the optical implementation of this method. One could imagine a reconstruction measured in intensity with a CCD camera and in phase with an interferometer. The study of different basic coding schemes other than that of Lee and the study of the evolution of the coding factor with time are also subjects for further investigation. Finally, there is still room for the improvement of the holograms obtained with a flip-flop method based on the Babinet principle.

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References


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