Complete test graph generation for symbolic real-time systems

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Abstract. We study the generation of test cases for symbolic real-time systems. By symbolic, we mean that the specification of the implementation under test (IUT) is described by using variables and parameters. And by real-time, we mean that the specification of the IUT contains timing constraints. Our method combines and generalizes two test methods presented in previous work, namely: 1) a method for generating test cases for (non-symbolic) real-time systems, and 2) a method for generating test cases for (non-real-time) symbolic systems.

Keywords: Test cases generation, real-time test, symbolic test, timed input output symbolic automata, test architecture.

1. Introduction

Conformance testing (or more simply, testing) aims at checking whether an implementation under test (IUT) conforms to a formal specification of the intended behavior of the IUT. Test activity consists of: generating test cases from the specification, and executing them on the IUT. We study the generation phase, but we also propose a test architecture for the execution phase. Among existing work on testing, we are interested in the following two complementary works:

Test of real-time systems (or real-time test): the specification of the IUT contains order and timing constraints of the interactions between the IUT and its environment. Several real-time test methods have been developed in the last years [Clarke, 1996, Springintveld et al., 1997, Braberman et al., 1997, Peleska et al., 1998, En-Nouaary et al., 1998, Nielsen, 2000, Cardell-Oliver, 2000a, Cardell-Oliver, 2000b, Khoumsi et al., 2003, Krichen and Tripakis, 2004].

Test of symbolic systems (or symbolic test): the specification of the IUT is described by using variables and parameters. A few symbolic test methods have been developed [Rusu et al., 2000, Rusu, 2002, Clarke et al., 2002]. These methods aim at avoiding the generation of test cases where all variables are instantiated. Note that symbolic techniques have also been developed in other areas than testing, e.g., model-checking [Behrmann et al., 2002] and diagnosis [Tripakis, 2002].

We propose a test generation method which combines real-time testing and symbolic testing. Our study is motivated by the desire to obtain a test cases generation method for real-time systems which does not require instantiation of variables (i.e., does not necessitate enumeration of their possible values). We first define the model of timed input output symbolic automata (Tiosa), which adds time to the IOSTS model of [Rusu et al., 2000] and is used to model the specification of the IUT. The approach of our test method is as follows:

*A longer version with proofs of lemmas and theorems is available in [Khoumsi, 2004a].
First, we express the test problem into a non-real-time form, by using a transformation of a T\textsubscript{iosa} into an automaton called Set-Exp-IOSA (SE\textsubscript{iosa}). SetExp denotes such a transformation, and SetExp(A) denotes the SE\textsubscript{iosa} obtained by transformation of a T\textsubscript{iosa} A. SetExp basically adds to the structure of a T\textsubscript{iosa} two additional types of actions: Set and Exp that model the setting and expiring of clocks, respectively. Then, we use and adapt the non-real-time symbolic test method of [Rusu et al., 2000].

As we will see, an advantage of our method is its simplicity because the main treatment of the real-time aspect is concentrated into one step.

The rest of the paper is structured as follows. Sect. 2 describes the T\textsubscript{iosa} model used to describe the specification of the IUT. In Sect. 3, we define formally the test problem to be solved. Sect. 4 introduces the SE\textsubscript{iosa} model and the transformation “SetExp : T\textsubscript{iosa} \mapsto SE\textsubscript{iosa}”. In Sect. 5, we propose a test architecture and present a theorem related to SetExp. Sect. 6 presents a method based on SetExp that solves the test problem. And in Sect. 7, we conclude the paper.

2. Timed IOSA (T\textsubscript{iosa})
In this section, we present timed input output symbolic automata (T\textsubscript{iosa}) used to describe the IUT and its specification. T\textsubscript{iosa} is a combination of timed automata of [Khoumsi et al., 2003] and input output symbolic transition systems (IOSTS) of [Rusu et al., 2000].

2.1. Clocks and related concepts
A clock \( c_i \) is a real variable whose value can be reset (to 0) with the occurrence of an action and such that, between two resets, its derivative (w.r.t. time) is equal to 1.

Let \( \mathcal{H} = \{c_1, \cdots, c_N\} \) be a set of clocks.

A Clock Guard (CG) is a conjunction of formula(s) in the form “\( c_i \sim k \)”, where \( c_i \in \mathcal{H}, \sim \in \{<, >, \leq, \geq, =\} \), and \( k \) is a nonnegative integer. A CG can be the constant True (empty conjunction). Let \( \Phi_{\mathcal{H}} \) be the set of CGs using clocks of \( \mathcal{H} \).

A clock reset is a subset of \( \mathcal{H} \), and \( 2^\mathcal{H} \) denotes the set of clock resets.

2.2. Data and related concepts
A variable is a data whose value can be set with the occurrence of an action. Let \( \mathcal{V} \) be a set of variables.

A constant is a data whose value is set once at initial time. Let \( \mathcal{C} \) be a set of constants.

A (communication) parameter is a data which is transmitted as a parameter of an action. Let \( \mathcal{P} \) be a set of parameters.

A Data Guard (DG) is a boolean expression using data of \( \mathcal{D} = \mathcal{V} \cup \mathcal{C} \cup \mathcal{P} \). Let \( \Gamma_{\mathcal{D}} \) denote the set of data guards (we consider that \( True \in \Gamma_{\mathcal{D}} \)).

A Variable Assignment (VA) is a set of assignments \( v := E \), where \( v \in \mathcal{V} \) and \( E \) is an expression depending on \( \mathcal{D} \). Let \( \Lambda_{\mathcal{D}} \) be the set of VAs.

The domain of definition of every \( x \in \mathcal{D} \) is noted Type\( (x) \).

2.3. Syntax of T\textsubscript{iosa}
The syntax of a T\textsubscript{iosa} is defined by \( (L, l_0, \mathcal{H}, \mathcal{D}, \mathcal{I}, \Sigma, \mathcal{T}) \), where: \( L \) is a finite set of locations, \( l_0 \) is the initial location, \( \mathcal{H} \) is a finite set of clocks, \( \mathcal{D} = \mathcal{V} \cup \mathcal{C} \cup \mathcal{P} \) is a finite set of data, \( \mathcal{I} \) is a boolean expression depending of \( \mathcal{V} \cup \mathcal{C} \) called initial condition, \( \Sigma \) is a finite set of actions, and \( \mathcal{T} \) is a transition relation. To each \( a \in \Sigma \) is associated a tuple \( (p_1, \cdots, p_k) \)
of parameters (possibly empty) denoted \( \theta_a \). Signature of \( a \) is denoted \( \text{Sig}(a) \) and defined as follows:

\[
\text{Sig}(a) = \begin{cases} 
\langle \text{Type}(p_1) \cdots \text{Type}(p_k) \rangle (\text{possibly empty tuple}) & \text{if } a \text{ is an input or output} \\
\text{empty tuple} & \text{if } a \text{ is an internal action}
\end{cases}
\]

There are three kinds of actions: the reception of an input \( i \) containing the tuple \( \theta_i \), written \( ?i(\theta_i) \); the sending of an output \( o \) containing the tuple \( \theta_o \), written \( !o(\theta_o) \); and the occurrence of an internal action \( a \), written \( e_a \). \( \theta_i \) and \( \theta_o \) are omitted when empty. Receptions of inputs and sending of outputs are observable actions, and occurrences of internal actions are unobservable actions. A transition of \( T_{\text{iosa}} \) is defined by \( Tr = \langle q; r; \sigma; \text{CG}; Z_\sigma; \text{DG}; \text{VA} \rangle \), where: \( q \) and \( r \) are origin and destination locations; \( \sigma \) is an action in the form \( ?i, !o \) or \( e_a \); \( \theta_\sigma \) is the (possibly empty) tuple of parameters associated to \( \sigma \); \( \text{CG} \) and \( Z_\sigma \) are a clock guard and a clock reset; and \( \text{DG} \) and \( \text{VA} \) are a data guard and a variable assignment defined in \( V \cup C \cup \theta_\sigma \).

The index \( \sigma \) in \( Z_\sigma \) means that the clock reset of a transition depends only on its action. This restriction guarantees determinizability of \( T_{\text{iosa}} \) [Khoumsi et al., 2003].

Fig. 1 illustrates the definition of \( T_{\text{iosa}} \) through an example. Locations are represented by nodes, and a transition \( Tr = \langle q; r; \sigma; \text{CG}; Z_\sigma; \text{DG}; \text{VA} \rangle \) is represented by an arrow linking \( q \) to \( r \) and labelled in 3 lines by: \( \sigma(\theta_\sigma) \), \( (CG; Z_\sigma) \) and \( (DG; VA) \). The \( \text{CG} \) and \( \text{DG} \) evaluate to \( \text{True} \) and the absence of \( Z_\sigma \) or \( \text{VA} \) are indicated by “-”. \( \Sigma = \{ \phi, \alpha, \beta, \rho \} \), \( \mathcal{H} = \{ c_1 \} \), \( \mathcal{V} = \{ x \} \), \( \mathcal{C} = \{ p \} \), \( \mathcal{P} = \{ m \} \), and \( x, p, m \) are integers. \( \phi \) cannot be an internal action because it contains parameter \( m \), and the other actions can be of any type.

2.4. Semantics of \( T_{\text{iosa}} \)

Let us define the semantics of a \( T_{\text{iosa}} \) \( A = (\mathcal{L}, l_0, \mathcal{H}, \mathcal{D}, \mathcal{I}, \Sigma, \mathcal{T}) \). At time \( \tau_0 = 0 \), \( A \) is at location \( l_0 \) with all clocks equal to 0, and variables and constants taking values such that \( \mathcal{I} \) evaluates to \( \text{True} \). The transition \( Tr = \langle q; r; \sigma; \text{CG}; Z_\sigma; \text{DG}; \text{VA} \rangle \) of \( A \) is enabled when \( q \) is the current location and both \( \text{CG} \) and \( \text{DG} \) evaluate to \( \text{True} \); otherwise, \( Tr \) is disabled. From this location \( q \), the action \( \sigma \) (containing parameters of \( \theta_\sigma \)) can be executed only when \( Tr \) is enabled; and after the execution of \( \sigma \) location \( r \) is reached, the clocks in \( Z_\sigma \) (if any) are reset, and the assignments in \( \text{VA} \) (if any) are applied. Before defining the semantics of \( T_{\text{iosa}} \) by a timed language, here are a few necessary definitions:

A timed action is a pair \((e, \tau)\) where \( e \) is an action and \( \tau \) is the instant of time when \( e \) occurs. When \( e \) is an input (resp. output, internal) action, then \((e, \tau)\) is called timed input (resp. timed output, timed internal) action.

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1Note that \( \text{DG} \) and \( \text{VA} \) of a transition \( Tr = \langle q; r; \sigma; \text{CG}; Z_\sigma; \text{DG}; \text{VA} \rangle \) are defined in \( V \cup C \cup \theta_\sigma \) and not in the whole \( D = V \cup C \cup P \).

2But when \( Tr \) is enabled, \( \sigma \) is not necessarily executed.
A timed sequence is a (finite or infinite) sequence of timed actions 

“(e₁, τ₁) · · · (eᵢ, τᵢ) · · ·”, where 0 < τ₁ < · · · < τᵢ < · · ·.

A timed trace is obtained from a timed sequence by removing all its timed internal actions.

Acceptance of a timed sequence \( \lambda^t = (e₁, \tau₁)(e₂, \tau₂) · · · \), for \( e₁, e₂, · · · \in \Sigma \). Let \( n \) be the length of \( \lambda^t \) (\( n \) can be infinite), and \( \lambda^t_i = (e₁, \tau₁) · · · (eᵢ, \tauᵢ) \) be the prefix of \( \lambda^t \) of length \( i \), for \( 0 \leq i \leq n \) (\( i \) is finite). \( \lambda^t \) is accepted by \( A \) iff \( \lambda^t \) is the empty sequence \( \lambda^t_0 \) or \( A \) has a sequence of length \( n \) of consecutive transitions \( Tr_1 Tr_2 · · · \) starting at \( t_0 \) and such that \( \forall i = 1, 2, · · · , n: \) the action of \( Tr_i \) is \( e_i \) and, after the execution of \( \lambda^t_{i−1} \), \( Tr_i \) is enabled at time \( \tau_i \). Intuitively, \( \lambda^t \) corresponds to an execution of \( A \).

Acceptance of a timed trace : Let \( \mu^t = (e₁, \tau₁)(e₂, \tau₂) · · · \) be a timed trace. \( \mu^t \) is accepted by \( A \) iff \( \mu^t \) is obtained by removing all the timed internal actions of a timed sequence accepted by \( A \). Intuitively, \( \mu^t \) corresponds to the observation of an execution of \( A \).

Definition 2.1 The Timed observable language of a \( \text{T}_{\text{iosa}} A \) \( \text{TOL}_{A}^\text{T_{iosa}} \) is the set of timed traces accepted by \( A \). That is, \( \text{TOL}_{A}^\text{T_{iosa}} \) models the observable behavior of \( A \).

Coming back to the example of Fig. 1, let \( \delta_{u,v} \) be the delay between actions \( u \) and \( v \):

- The \( \text{T}_{\text{iosa}} \) is initially in location \( l_0 \). At the occurrence of \( \phi(m) \), location \( l_1 \) is reached and variable \( x \) is assigned with the value of \( m \).
- From \( l_1 \), the \( \text{T}_{\text{iosa}} \) reaches \( l_2 \) at the occurrence of \( \alpha \).
- From \( l_2 \), the \( \text{T}_{\text{iosa}} \) reaches \( l_3 \) or \( l_4 \) at the occurrence of \( \beta \). \( l_3 \) is reached only if \( \delta_{\alpha,\beta} < 3 \) and \( x \geq p \), and \( l_4 \) is reached only if \( \delta_{\alpha,\beta} > 2 \) and \( x \leq p \).
- We see that there is a nondeterminism when \( 2 < \delta_{\alpha,\beta} < 3 \) and \( x = p \). \( x \) is incremented when \( l_4 \) is reached.
- From \( l_3 \), the \( \text{T}_{\text{iosa}} \) executes nothing.
- From \( l_4 \), the \( \text{T}_{\text{iosa}} \) reaches \( l_1 \) at the occurrence of \( \rho \). We have \( \delta_{\alpha,\rho} > 3 \).

The class of \( \text{T}_{\text{iosa}} \) that we will consider obeys to the following hypothesis:

Hypothesis 2.1 Infinite timed sequences accepted by a \( \text{T}_{\text{iosa}} A \) are non-zeno, i.e., an infinite number of actions cannot be executed into a finite time interval.

Remark 2.1 Unlike [Alur, 1999], with our model consecutive actions cannot occur at the same time. Actually this is not a restriction, because we consider that if an action \( e \) is followed an action \( f \), then \( e \) and \( f \) are not simultaneous.

3. Test problem to be solved

3.1. Conformance relation between \( \text{T}_{\text{iosa}} \), and some related lemmas

In the following: \( I \) and \( S \) denote two \( \text{T}_{\text{iosa}} \) over the same alphabet \( \Sigma \), and \( o \) is an output action of \( \Sigma \). We define the following conformance relation:

Definition 3.1 \( I \text{ conf}_{\text{T_{iosa}}} S \) means, \( \forall \lambda \in \text{TOL}_{S}^\text{T_{iosa}}: \)

\[(\lambda \cdot (o, \tau) \in \text{TOL}_{S}^\text{T_{iosa}}) \Rightarrow (\lambda \cdot (o, \tau) \in \text{TOL}_{I}^\text{T_{iosa}}).\]

If the \( \text{IUT} \) is modelled by \( I \), “\( I \text{ conf}_{\text{T_{iosa}}} S \)” means that after an execution of the \( \text{IUT} \) accepted by \( S \), the \( \text{IUT} \) can generate an output \( o \) at time \( \tau \) only if \( S \) accepts \( o \) at time \( \tau \).

Let \( \Sigma_I \) be the set of inputs of the alphabet \( \Sigma \), and \( \text{Univ} \) be a “universal” \( \text{T}_{\text{iosa}} \) accepting all the timed traces over \( \Sigma \). That is, \( \text{TOL}_{\text{Univ}} \) contains every timed trace over \( \Sigma \). The following definition is inspired from [Jard et al., 1999, Jéron et al., 2003].
Definition 3.2 The input-completion of a $T_{\text{iosa}} A = (L, I_0, H, D, I, \Sigma, T)$ is a $T_{\text{iosa}}$ $\text{InpComp}(A)$ that contains all the timed traces of $A$, as well as all the timed traces that diverge from the timed traces of $A$ by executing inputs not accepted by $A$. Formally, $\text{InpComp}(A)$ is a $T_{\text{iosa}}$ such that:

$$TOL^{T_{\text{iosa}}}_{\text{InpComp}}(A) = TOL^{T_{\text{iosa}}}_A \cup \bigcup_{w \in TOL^{T_{\text{iosa}}}_A, a \in \Sigma} a \cdot w^*.$$ 

A is said input-complete iff $A = \text{InpComp}(A)$. Intuitively, an input-complete $T_{\text{iosa}}$ accepts every input at any time.

Lemma 3.1 $(I \text{con}f^{T_{\text{iosa}}}_{IUT} S) \Leftrightarrow (I \text{con}f^{T_{\text{iosa}}}_{\text{InpComp}} S))$.

Lemma 3.1 implies that we can replace a $T_{\text{iosa}} S$ by its input-completion before checking whether a $T_{\text{iosa}} I$ conforms to it, w.r.t. $\text{con}f^{T_{\text{iosa}}}_{IUT}$. However, Def. 3.2 is not constructive and we do not know how to compute $\text{InpComp}(S))$ from a $T_{\text{iosa}} S$ in the general case. Hence, we will use the following hypothesis:

Hypothesis 3.1 In “$I \text{con}f^{T_{\text{iosa}}}_{IUT} S$”, we assume $S$ input-complete.

Note that Lemma 3.1 and Hyp. 3.1 are inspired from their non-real-time and non-symbolic (i.e., without clocks and data) version in [Jard et al., 1999].

Lemma 3.2 With Hypothesis 3.1: $I \text{con}f^{T_{\text{iosa}}}_{IUT} S \Leftrightarrow TOL^{T_{\text{iosa}}}_I \subseteq TOL^{T_{\text{iosa}}}_S$.

Lemma 3.2 means that with Hypothesis 3.1, $\text{con}f^{T_{\text{iosa}}}_{IUT}$ is simplified into an inclusion of timed observable languages of $T_{\text{iosa}}$.

3.2. Test purpose, and test hypothesis

Definition 3.3 A $T_{\text{iosa}} A = (L, I_0, H, D, I, \Sigma, T)$ is said to be complete iff: $\forall l \in L$ and $\forall e \in \Sigma$, $e$ is enabled in $l$ for every possible clock value and data value. Intuitively, a complete $T_{\text{iosa}} A$ accepts every (input, output or internal) action at any time.

Definition 3.4 A trap location $q$ is a location in which, for each event $\sigma \in \Sigma$, there is a selfloop transition $Tr = \langle q; \sigma; q; True; \emptyset; True; \emptyset \rangle$. That is, when a trap is reached then it is never left and every action is executable from it at any time.

Definition 3.5 A test purpose is a $T_{\text{iosa}} TP$ used to select the behaviors to be tested. By analogy with [Jard and Jéron, 2002, Rusu et al., 2000, Khoumsi et al., 2003], TP is complete, deterministic, and equipped with two sets of trap locations $A$ and $R$ (for Accept and Refuse). Timed Sequences to be considered in testing activity are those terminating in and not traversing a location $A$, while timed sequences to be ignored are those traversing or terminating in a location $R$.

A test purpose is used to select a part of the specification (and thus, to ignore the remaining part) before applying a test generation method.

The following test hypothesis is inspired from [Tretmans, 1992]:

Hypothesis 3.2 $IUT$ can be described by a (possibly unknown) input-complete $T_{\text{iosa}} IUT$.

3.3. Formalization of the test problem

Given two $T_{\text{iosa}}$ $Spec$ and $TP$ over the same alphabet, describing the specification and the test purpose, respectively, the objective is to synthesize an automaton $CTG$ (Complete Test Graph), from which test cases can be extracted and executed in order to determine whether: $IUT \text{con}f^{T_{\text{iosa}}}_{IUT} Spec$. We assume $Spec$ input-complete (see Hyp. 3.1). $CTG$ is an interesting automaton because it contains all test cases of $Spec$ corresponding to $TP$.

The test system takes into account $TP$ by ignoring every execution $\lambda$ of the $IUT$ accepted by $Spec$ (i.e., $\lambda \in TOL^{T_{\text{iosa}}}_{IUT} \cap TOL^{T_{\text{iosa}}}_{SpecTP}$) and such that: a location $R$ of $TP$ may be reached by $\lambda$, or no location $A$ of $TP$ is reachable after $\lambda$ by $Spec$. 

4. Transformation of T_{iosa} into SE_{iosa}

A transformation, called SetExp, is presented in [Khoumsi and Ouedraogo, 2004] and applied in [Khoumsi, 2002a, Khoumsi et al., 2003, Khoumsi and Nourelfath, 2002, Khoumsi, 2002b, Khoumsi, 2004b]. SetExp basically transforms a timed automaton (TA) into a finite state automaton by adding to the structure of the TA two additional types of actions: Set and Exp, that capture the temporal aspect of the TA. In the present article, we apply SetExp to T_{iosa} instead of TA. When applying SetExp to T_{iosa}, data and their DG and VA are processed just like action labels because their semantics is ignored. The latter is taken into account when using (interpreting, processing, . . .) the automaton called SE_{iosa} that results from SetExp. Our test problem will be solved in Sect. 6 by using SetExp.

In this Section, we present the SE_{iosa} model and illustrate SetExp by an example (a detailed description of SetExp can be found in [Khoumsi and Ouedraogo, 2004]). Let A be a T_{iosa} over an alphabet Σ and SetExp(A) be the SE_{iosa} obtained by transformation.

4.1. Actions Set and Exp

Set(c_i, k) means: clock c_i is reset (to 0) and will expire when its value is equal to k. More generally, Set(c_i, k_1, k_2, . . . , k_p) means that c_i is reset and will expire several times, when its value is equal to k_1, k_2, . . . , k_p, resp. We assume without loss of generality that k_1 < k_2 < . . . < k_p.

Exp(c_i, k) means: clock c_i expires and its current value is k.

Therefore, Set(c_i, k) is followed (after a delay k) by Exp(c_i, k), and Set(c_i, k_1, k_2, . . . , k_p) is followed (after delays k_1, . . . , k_p) by Exp(c_i, k_1), Exp(c_i, k_2), . . . , Exp(c_i, k_p). When a Set(c_i, m) occurs, then all Exp(c_i, *) which were expected before this Set(c_i, m) are cancelled.

4.2. Basic principle of SetExp

In a T_{iosa} A, a clock c is reset with the objective to compare later its value to (at least) one constant, say k. The action Set(c, k) is very convenient for that purpose, because it resets c and programs Exp(c, k) which is a notification when c = k. The principle of SetExp when applied to a T_{iosa} A is:

1. To replace each clock reset in A by the appropriate Set action; and then
2. To construct a finite state automaton, denoted SetExp(A), that accepts sequences containing actions of A and Set actions obtained in (1) and the corresponding Exp actions, and such that the order of actions in each accepted sequence respects order and timing constraints of A.

In order to give a trivial example that illustrates SetExp, let us consider the following two expressions. Expression 1: a task must be realized in less than two units of time. Expression 2: at the beginning of the task an alarm is programmed for occurring after two time units, and the task must be terminated before the alarm. It is clear that these two expressions define the same timing constraint. In this example, SetExp can be used to obtain the second expression from the first one. The programming of the alarm corresponds to a Set action, and the occurrence of the alarm corresponds to an Exp action.

4.3. Transitions of SE_{iosa}

Recall that a transition of T_{iosa} is defined by ⟨q; r; σ; θ_σ; CG; Z_σ; DG; VA⟩ and represented in a figure by an arrow linking q to r and labelled by: σ(θ_σ), (CG; Z_σ) and (DG; VA). Let: η denote an action of the alphabet Σ of the T_{iosa} A with its parameters, S (resp. E) denote a set of Set (resp. Exp) actions, and occurrence of S (resp.
\( \mathcal{E} \) mean the simultaneous occurrences of all the actions in \( \mathcal{S} \) (resp. \( \mathcal{E} \)). We categorize transitions of \( \text{SE}_{\text{iosa}} \) \( \text{SetExp}(A) \) into three types:

**Type 1** : transition labelled by (\( \mathcal{E} \)) represents the occurrence of \( \mathcal{E} \).

**Type 2** : transition labelled by (\( \eta \)) or (\( \eta, \mathcal{S} \)), and by a \( \text{DG} \) and a \( \text{VA} \). (\( \eta \)) represents the occurrence of \( \eta \), (\( \eta, \mathcal{S} \)) represents the simultaneous occurrences of \( \eta \) and \( \mathcal{S} \), and \( \text{DG} \) and \( \text{VA} \) have the same semantics as in \( \text{T}_{\text{iosa}} \). A transition \( \text{TR} \) of Type 2 in \( \text{SetExp}(A) \), corresponds to a transition \( \text{Tr} \) of \( A \) such that: \( \text{Tr} \) and \( \text{TR} \) have the same \( \eta \) and \( \text{DG} \) and \( \text{VA} \), and \( \text{Tr} \) resets the clocks in the \( \mathcal{S} \) (if any) of \( \text{TR} \).

**Type 3** : transition labelled by (\( \mathcal{E}, \eta \)) or (\( \mathcal{E}, \eta, \mathcal{S} \)), and by a \( \text{DG} \) and a \( \text{VA} \). (\( \mathcal{E}, \eta \)) represents the simultaneous occurrences of \( \mathcal{E} \) and \( \eta \), and (\( \mathcal{E}, \eta, \mathcal{S} \)) represents the simultaneous occurrences of \( \mathcal{E}, \eta \) and \( \mathcal{S} \). A transition \( \text{TR} \) of Type 3 in \( \text{SetExp}(A) \) corresponds to simultaneous executions of \( \mathcal{E} \) and a transition \( \text{Tr} \) of \( A \) such that: \( \text{Tr} \) and \( \text{TR} \) have the same \( \eta \) and \( \text{DG} \) and \( \text{VA} \), and \( \text{Tr} \) resets the clocks in the \( \mathcal{S} \) (if any) of \( \text{TR} \).

**Remark 4.1** A transition of type 3 corresponds to the simultaneity of two transitions of type 1 and 2, respectively.

**Definition 4.1** An \( \text{Exp-Trans} \) of \( \text{SetExp}(A) \) is a transition of type 1 or 3, i.e., whose label contains one or several \( \text{Exp} \) actions.

### 4.4. Example of transformation \( \text{SetExp} : \text{T}_{\text{iosa}} \mapsto \text{SE}_{\text{iosa}} \)

For the \( \text{T}_{\text{iosa}} \) \( A \) of Fig. 1, we obtain the \( \text{SE}_{\text{iosa}} \) \( \text{SetExp}(A) \) of Fig. 2, where \( \text{Set}_{2,3} \) is an abbreviation of \( \text{}^?\text{Set}(c_1, 2, 3) \), \( \text{Exp}_i \) is an abbreviation of \( \text{}^!\text{Exp}(c_1, i) \) for \( i = 2, 3 \), \( x++ \) means “\( x \) is incremented by 1”, and the constant \( \text{DG True} \) and the absence of \( \text{VA} \) are indicated by “-”. Transitions of Type 1 are those labelled \( \text{Exp}_i \). Transitions of Types 2 and 3 are labelled in two lines, where Line 2 consists of \( \text{}^!\text{DG; VA} \). Transitions of Type 2 are those labelled \( \phi(m) \), \( (\alpha, \text{Set}_{2,3}) \), \( \beta \) or \( \rho \) in Line 1. Transitions of Type 3 are those labelled \( \text{}^!\text{Exp}_i, \beta \) in Line 1, and correspond to the simultaneous executions of \( \text{Exp}_i \) and \( \beta \). We do not indicate whether each action \( \phi(m) \), \( \alpha \), \( \beta \) or \( \rho \) is an input, an output or an internal action, because this is irrelevant for the comprehension of \( \text{SetExp} \).

Note that this example does not respect Hyp. 3.1 (input-completeness), because the aim here is to illustrate \( \text{SetExp} \) while input-completeness is required by the test method and not by \( \text{SetExp} \). Note that clocks are real variables although they are compared to integers (2, 3), the latter being considered just as particular reals. For example, before the occurrence of \( \text{}^!\text{Exp}(c_1, 2) \), \( c_1 \) takes all the nonnull real values smaller than 2.

![Figure 2: SE_{iosa} SetExp(A) obtained from the T_{iosa} A of Fig. 1](image-url)
4.5. Syntax of SE$_{iosa}$

Let $A = (\mathcal{L}, l_0, \mathcal{H}, D, I, \Sigma, T)$ be a T$_{iosa}$ and $B = \text{SetExp}(A)$ be the corresponding SE$_{iosa}$. The syntax of $B$ can be defined by $B = (Q, q_0, D, I, \Lambda, \Psi)$, where: $Q$ is a finite set of states, $q_0$ is the initial state, $\Lambda$ is a finite alphabet that labels the transitions of $B$, $\Psi$ is a transition relation, and $D$ and $I$ are the same as those used in the definition of $A$. A transition of $B$ is syntactically defined by $\text{TR} = (q; r; \mu; DG; VA)$, where: $q$ and $r$ are origin and destination states; $\mu$ is the action(s) of TR; and $DG$ and $VA$ are a data guard and a variable assignment. $DG$ and $VA$ are always empty for transitions of Type 1 (see Sects. 4.3 and 4.4). $\Lambda$ is an alphabet consisting of labels of transitions of types 1, 2 and 3 (see Sect. 4.3).

4.6. Semantics of SE$_{iosa}$

Let us define the semantics of a SE$_{iosa}$ $B = (Q, q_0, D, I, \Lambda, \Psi)$. Initially, $B$ is at state $q_0$ with all clocks of $\mathcal{H}$ equal to 0, and variables and constants taking values such that $I$ evaluates to true. The transition $\text{TR} = (q; r; \mu; DG; VA)$ is enabled when $q$ is the current state and $DG$ (if any) evaluates to true; otherwise, TR is disabled. From this state $q$, $\mu$ (consisting of one or more actions) is executed only when TR is enabled; and after the execution of $\mu$: State $r$ is reached, and the assignments in VA (if any) are applied.

Let a sequence of SE$_{iosa}$ denote a sequence “$E_1E_2\cdots$”, where $E_1, E_2, \cdots \in \Lambda$; and let a trace of SE$_{iosa}$ be obtained from a sequence of SE$_{iosa}$ by removing all its internal actions. Let us define the semantics of a SE$_{iosa}$ $B = (Q, q_0, D, I, \Lambda, \Psi)$ by the set of sequences and traces accepted by $B$: 

**Acceptance of a (finite or infinite) sequence** $\lambda = E_1E_2\cdots$, for $E_1, E_2, \cdots \in \Lambda$. Let $n$ be the length of $\lambda$ ($n$ can be infinite), and $\lambda_i = E_1E_2\cdots E_i$ be the prefix of $\lambda$ of length $i$, for $0 \leq i \leq n$ ($i$ is finite). $\lambda$ is accepted by $B$ iff $\lambda$ is the empty sequence $\mu_0$ or:

- there exists a sequence of transitions $Tr_1Tr_2\cdots$ of $B$ of length $n$ such that $\forall i = 1,2,\cdots, n$: $Tr_i$ is labelled by $E_i$ and, after the execution of $\lambda_{i-1}$, $Tr_i$ is enabled.

Intuitively, $\lambda$ corresponds to an execution of $B$.

**Acceptance of a trace** $\mu$ : $\mu$ is accepted by $B$ iff $\mu$ is obtained by removing the internal actions of a sequence accepted by $B$. Intuitively, $\mu$ corresponds to the observation of an execution of $B$.

We can now introduce the notion of Observable Language of a SE$_{iosa}$:

**Definition 4.2** The observable language of a SE$_{iosa}$ $B$ ($OL_B^{SE_{iosa}}$) is the set of traces accepted by $B$. That is, $OL_B^{SE_{iosa}}$ models the observable behavior of $B$.

Given a SE$_{iosa}$ $B$, one can remark that $OL_B^{SE_{iosa}}$ implicitly respects the following condition, called the Consistency condition: every $\text{Set}(c,k)$ and its corresponding $\text{Exp}(c,k)$ are effectively separated by time $k$.

We terminate this section by presenting a fundamental property of $\text{SetExp}$. Let $TL = \text{AddTime}(L)$ be a timed language obtained from a language $L$ by associating a time to each action such that the consistency condition is respected. Let $\text{RmvSetExp}(TL)$ be obtained from a timed language $TL$ by removing all the $\text{Set}$ and $\text{Exp}$ actions, if any. We have the following theorem of equivalence:

**Theorem 4.1** $\text{RmvSetExp}(\text{AddTime}(OL_{\text{SetExp}(A)})) = TOL_A^{T_{iosa}}$.

Intuitively, Theorem 4.1 states that from a behavioral point of view, there is no difference between $A$ and $\text{SetExp}(A)$ for an observer who does not see (or ignores) $\text{Set}$ and $\text{Exp}$ actions. In a sense, $\text{SetExp}(A)$ does nothing but add some new actions ($\text{Set}$ and $\text{Exp}$) to $A$ that capture the relevant temporal aspect of $A$. As we will see in the next section, in our test method, these $\text{Set}$ and $\text{Exp}$ are physical actions that are produced by the test system.
5. Test architecture, and a theorem

Given two \( \text{T}_{\text{iosa}} \)s \( \text{Spec} \) and \( \text{TP} \) over the same alphabet, recall that the objective is to synthesize an automaton \( \text{CTG} \), from which test cases can be extracted and executed in order to determine whether the \( \text{IUT} \) conforms to the part of \( \text{Spec} \) corresponding to \( \text{TP} \). \( \text{CTG} \) will not be directly computed on the \( \text{T}_{\text{iosa}} \)s \( \text{Spec} \) and \( \text{TP} \), but rather on a \( \text{SE}_{\text{iosa}} \) computed from the two \( \text{T}_{\text{iosa}} \)s. In order to make the link between \( \text{CTG} \) and the \( \text{IUT} \), we use a particular test architecture [Khoumsi et al., 2003] that we now present. It is represented in Fig. 3:

**Clock-Handler** receives \( \text{Set} \) events and sends \( \text{Exp} \) actions. (It respects the consistency condition.)

**Test-Controller** sends inputs to the \( \text{IUT} \), receives outputs from the \( \text{IUT} \), sends \( \text{Set} \) actions to Clock-Handler, and receives \( \text{Exp} \) actions from Clock-Handler.

We define the following conformance relation \( \text{conf}_{\text{SE}_{\text{iosa}}} \) which is simply an inclusion of observable languages of \( \text{SE}_{\text{iosa}} \):

**Definition 5.1** Let \( I' \) and \( S' \) be two \( \text{SE}_{\text{iosa}} \)s over the same alphabet:

\[
(I' \ \text{conf}_{\text{SE}_{\text{iosa}}} \ S') \iff (\text{OL}_{I'}^{SE_{\text{iosa}}} \subseteq \text{OL}_{S'}^{SE_{\text{iosa}}}).
\]

We have the following theorem, where \( \text{SUT} \) consists of the \( \text{IUT} \) and Clock-Handler:

**Theorem 5.1** Let \( S \) be an input-complete \( \text{T}_{\text{iosa}} \). If Test-Controller generates \( \text{Set} \) actions only when they are accepted by \( \text{SetExp}(S) \), then:

\[
(\text{IUT conf}_{\text{SE}_{\text{iosa}}} \ S) \iff (\exists \ \text{SE}_{\text{iosa}} \ \text{SUT accepting behavior of SUT such that SUT conf}_{\text{SE}_{\text{iosa}}} \ \text{SetExp}(S)).
\]

The above theorem implies that we can check “\( \text{SUT conf}_{\text{SE}_{\text{iosa}}} \ \text{SetExp}(S) \)” instead of “\( \text{IUT conf}_{\text{SE}_{\text{iosa}}} \ S \)”. We have transformed the test of a real-time symbolic system into a non-real-time form, and thus, we can (and will) use and adapt a non-real-time method of Symbolic Test Generation (STG) [Rusu et al., 2000].

**Figure 3: Test architecture**

This architecture is applicable only if transitions executing internal (i.e., unobservable) actions do not reset clocks. In fact, in order to generate \( \text{Set} \) actions, Test-Controller needs to observe every action to which is associated a clock reset. Hence the following hypothesis:

**Hypothesis 5.1** Transitions executing internal actions do not reset clocks.

Hyp. 5.1 means that there is no timing constraint relatively to internal actions.
6. Method of test generation

Our test method combines two complementary test methods presented in [Rusu et al., 2000] and [Khoumsi et al., 2003], respectively. It consists of four steps outlined in Fig. 4 and described in subsections 6.1 to 6.4. Its inputs are \( \text{Spec} \) (input-complete, from Lemma 3.1 and Hyp. 3.1) and \( \text{TP} \) (complete, from Def. 3.5). In a first step, we compute a \( T_{\text{iosa}} \text{SpecTP} \) which accepts (all and only) the timed sequences of \( \text{Spec} \) and indicates the locations corresponding to the locations \( A \) and \( R \) of \( \text{TP} \). Then, we synthesize in three steps (2 to 4) a complete test graph (\( \text{CTG} \)), from which a set of test cases can be extracted and executed on the \( \text{IUT} \) in order to determine whether: \( \text{IUT} \text{conf} T_{\text{iosa}} \text{SpecTP} \). The indication \( A \) and \( R \) is used to ignore every execution of the \( \text{IUT} \) that leads to a location \( R \) or from which no location \( A \) is reachable. (See Sects. 3.2 and 3.3 for more detail.) The fact that \( \text{TP} \) is deterministic and complete implies that \( \text{Spec} \) is input-complete iff \( \text{SpecTP} \) is input-complete.

An advantage of our method is its simplicity because the main treatment of the real-time aspect is concentrated in Step 2. Steps 1, 3 and 4 constitute a slight adaptation of the (non-real-time) symbolic test generator (STG) [Rusu et al., 2000].

\[
\text{Synchronized product} \quad \text{SpecTP} \quad \text{SetExp} \quad \text{SpecTP}_\text{SEA} \quad \text{Extraction of visible behaviour} \quad \text{SpecTP}_\text{VIS} \quad \text{Computation of complete test graph} \quad \text{CTG}
\]

\textbf{Figure 4: Steps of the test method}

Figure 5 represents \( \text{Spec} \) and \( \text{TP} \) over the alphabet \( \Sigma = \{ ?\phi, ?\sigma, !\rho, \epsilon_a, \epsilon_b \} \) used to illustrate the steps of the test method. Data of \( \text{Spec} \) are \( \mathcal{H}^1 = \{ c_1 \}, \mathcal{V}^1 = \{ x \}, \mathcal{C}^1 = \{ p \}, \mathcal{P}^1 = \{ m \} \), where \( x, p, m \) are integers. Data of \( \text{TP} \) are \( \mathcal{H}^2 = \mathcal{V}^2 = \mathcal{C}^2 = \emptyset, \mathcal{P}^2 = \{ n \} \), where \( n \) is integer. \( \neq x \) means any action of \( \Sigma \) different from \( x \), and \( ?* \) means any input \( \in \Sigma \) (i.e., \( ?\phi \) or \( ?\sigma \)). \( \text{Spec} \) was not initially input-complete and we represent by dotted arrows the part that has been added to make \( \text{Spec} \) input-complete (see Hyp. 3.1). Transitions labelled only by an action mean that: their (clock and data) guards are equal to the constant \( \text{True} \), and they do not reset clocks and do not have variable assignments. The \( \text{TP} \) of this example means that: we are interested to test executions of \( \text{Spec} \) terminating by the first occurrence of \( !\rho \) without traversing Location \( TL \).

This example of \( \text{TP} \) is taken very simple (with no timing constraint) in order to clarify the operations of the different steps. Even in a concrete case, \( \text{TP} \) should be relatively simple because the objective of its use is to select a relatively small part of the specification in order to concentrate only in certain aspects of the specification.

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3 Actually, STG is a software tool. But here, STG denotes the theoretical method that underlies the tool.
6.1. Step 1: Synchronous product of $Spec$ and $TP$

The aim is to compute a $T_{iosa} \ SpecTP$ equivalent to $Spec$ such that locations of $SpecTP$ that correspond to locations $A$ (resp. $R$) of $TP$ are also denoted $A$ (resp. $R$). For that purpose, we need to define the synchronized product of two $T_{iosa}$s.

Let $A^i = (L^i, l^i_0, \mathcal{H}^i, D^i, I^i, \Sigma^i, T^i)$ where $D^i = V^i \cup C^i \cup P^i$, for $i = 1, 2$, be two $T_{iosa}$s. The synchronized product of $A^1$ and $A^2$, written $A^1 \otimes A^2$, is inspired (but different) from the synchronized product of TA [Alur and Dill, 1994] and the synchronized product of IOSTS [Rusu et al., 2000]. $A_1 \otimes A_2$ is defined iff the following four conditions are satisfied:

1. $\Sigma^1 = \Sigma^2$. The common alphabet will then be denoted $\Sigma$. This condition can be easily relaxed [Rusu et al., 2000], but we will keep it for simplicity.
2. $\mathcal{H}^1 \cap \mathcal{H}^2 = \emptyset$ [Alur and Dill, 1994].
3. $(V^1 \cup P^1) \cap (V^2 \cup P^2) = \emptyset$, $(C^1 \cup P^1) \cap P^2 = \emptyset$, and $(C^2 \cup P^2) \cap P^1 = \emptyset$ [Rusu et al., 2000].
4. Each action $\alpha \in \Sigma$ has the same signature in $A^1$ and $A^2$ [Rusu et al., 2000].

Assuming the above four conditions satisfied, $A^1 \otimes A^2$ is defined by $(L, l_0, \mathcal{H}, D, I, \Sigma, T)$ such that: $L = L^1 \times L^2$, $l_0 = (l^1_0, l^2_0)$, $\mathcal{H} = \mathcal{H}^1 \cup \mathcal{H}^2$, $D = V \cup C \cup P$, $\mathcal{V} = V^1 \cup V^2$, $C = (C^1 \cup C^2) \setminus V$, $P = P^1 \cup P^2$, $I = (I^1 \cap I^2)$, and the set of transitions $T$ is defined as follows:

For each pair of transitions $((q^i; r^i; \sigma; \theta^i_\alpha; CG^i; Z^i_\alpha; DG^i; VA^i)) \in T^i$, $i = 1, 2$:

**If $\theta^1_\alpha$ and $\theta^2_\alpha$ are the empty tuple $\epsilon$** : then there is a transition

$$((q^1; q^2); (r^1; r^2); \sigma; \epsilon; CG^1 \land CG^2; Z^1_\alpha \cup Z^2_\alpha; DG^1 \land DG^2; VA^1 \cup VA^2) \in T.$$

**If $\theta^1_\alpha$ and $\theta^2_\alpha$ are not empty** : let $DG^{1,2}$ (resp. $VA^{1,2}$) denote the expression obtained by replacing in $DG^2$ (resp. $VA^2$) each parameter from $\theta^2_\alpha$ by the corresponding, same-position parameter from $\theta^1_\alpha$; then there is a transition

$$((q^1; q^2); (r^1; r^2); \sigma; \theta^1_\alpha; CG^1 \land CG^2; Z^1_\alpha \cup Z^2_\alpha; DG^1 \land DG^{1,2}; VA^1 \cup VA^{1,2}) \in T.$$  

This procedure is inspired from [Rusu et al., 2000].

In Step 1, we compute $SpecTP = Spec \otimes TP$, whose locations that correspond to locations $A$ (resp. $R$) of $TP$ are indicated by $A$ (resp. $R$). Completeness of $TP$ implies that $Spec$ and $SpecTP$ are observationally equivalent (i.e., $TOL_{SpecTP}^{T_{iosa}} = TOL_{Spec}^{T_{iosa}}$). Completeness of $TP$ and input-completeness of $Spec$ imply that $SpecTP$ is input-complete. The effect of $Spec \otimes TP$ is to determine in $Spec$ all the executions that correspond to locations $A$ and $R$, respectively.
For Spec and TP of Fig. 5, we obtain the SpecTP of Fig. 6. Locations $L_1$ and $A_1$ are equivalent in the sense that the same behavior can be produced from them. The difference between these two locations is that only $A_1$ corresponds to Location $A$ of TP. Note that, in accordance with the definition of synchronized product, parameter $n$ of TP has been removed by replacing it by parameter $m$ of Spec.

![Figure 6: Step 1: SpecTP obtained from Spec and TP of Fig. 5](image)

6.2. Step 2 : Transforming the $T_{iosa}$ SpecTP into $SE_{iosa}$

We transform the problem into a non-real-time form by computing $SpecTP_{SE_{iosa}}^{SE_{iosa}} = SetExp(SpecTP)$. For the SpecTP of Fig. 6, we obtain the $SpecTP_{SE_{iosa}}^{SE_{iosa}}$ of Fig. 7. $Set_{i, j}$ denotes $?Set(c_1, 2, 3)$, and $Exp_i$ denotes $!Exp(c_1, i)$. States of $SpecTP_{SE_{iosa}}^{SE_{iosa}}$ that correspond to location $A$ (resp. $R$) of SpecTP are indicated by $A$ (resp. $R$). $(Exp_i, \Sigma)$ means the simultaneous occurrences of $Exp_i$ and any $x \in \Sigma$, and nodes linked by a broken line indicate the same state. State $A_1$ of Fig. 7 is equivalent to State $S_1$ with the difference that $S_1$ does not correspond to a location $A$ of TP.

In Fig. 7 and subsequent figures, if $DG$ evaluates to $true$ and $VA$ is empty in a transition, then $(DG; VA)$ is not represented.

![Figure 7: Step 2: SpecTP_{SE_{iosa}}^{SE_{iosa}} obtained from SpecTP of Fig. 6](image)

6.3. Step 3 : Extracting the visible behavior of $SpecTP_{SE_{iosa}}^{SE_{iosa}}$

We construct the visible behavior of $SpecTP_{SE_{iosa}}^{SE_{iosa}}$ in three substeps:

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4They are duplicated for the sake of clarity.
Substep 3a: Internal actions are eliminated by projection into the observable alphabet. For that purpose, we can adapt a procedure proposed in [Rusu et al., 2000]. The result is denoted Vis(SpecTPSEiona). The adaptation consists of a preliminary step where internal actions in transitions of Type 3 are simply erased. After that, we can use the procedure of [Rusu et al., 2000] because the remaining internal actions are “alone” in their transitions. (Recall that we consider only the case where internal actions do not reset clocks.)

Substep 3b: Vis(SpecTPSEiona) is determinized by using a heuristic proposed in [Rusu et al., 2000]. The result is denoted Det(Vis(SpecTPSEiona)).

Substep 3c: Note that every state of Det(Vis(SpecTPSEiona)) corresponds to one or several states of SpecTPSEiona. States R and A of Det(Vis(SpecTPSEiona)) are selected as follows:

- We call R every state corresponding to at least one state R of SpecTPSEiona. Intuitively, we ignore every execution which can correspond to a sequence not to be tested.
- We can call A every state corresponding to no state R and at least one state A of SpecTPSEiona. Intuitively, we accept every execution which: 1) cannot correspond to a sequence not to be tested, and 2) can correspond to a sequence to be tested. Another way, which seems more realistic, is to call A every state corresponding only to states A of SpecTPSEiona. Intuitively, we accept an execution only when we are sure that it corresponds to a sequence to be tested.

The result is denoted SpecTPVISSEiona.

For the SpecTPSEiona of Fig. 7, we obtain SpecTPVISSEiona of Fig. 8.

6.4. Step 4: Computing a Complete Test Graph (CTG)

Recall that a transition of SEiona is labelled in the form (E) or in one of the following four forms: (σ), (σ, S), (E, σ), (E, σ, S), in addition to (DG; VA). Let “output transition of SEiona” denote any transition labelled in one of the four forms such that σ is an output of the IUT. By analogy with [Jard and Jéron, 2002, Khoumsi et al., 2003, Rusu et al., 2000], we construct a Complete Test Graph (CTG) as follows:

- Let L2A be the set of states of SpecTPVISSEiona from which a A is accessible.
- Let Pass denote the set of states A of SpecTPVISSEiona.
- Let Fail = {fail} consist of a new state that is reached by every non-specified output transition of SpecTPVISSEiona executable from L2A.
- Let Inconc be the set of states of SpecTPVISSEiona that are not in L2A ∪ Pass and are accessible from L2A by a single output transition of SpecTPVISSEiona.
- We then obtain CTG from SpecTPVISSEiona by:
  - adding (implicitly) state Fail and its incoming transitions,
  - removing every state ⊊ L2A ∪ Pass ∪ Inconc ∪ Fail, and
  - removing outgoing transitions of every state ⊊ Pass ∪ Inconc.
To synthesize test sequences executable in acceptable time (that is, to avoid that Test-
Controller waits for an input during a very long time), we select a delay \( T \) and define a fictitious event \( \delta \) whose occurrence means: no observable action occurs during a period equal to \( T \). We then proceed as follows:

- we define a new state \( \text{inconc}_\delta \in \text{Inconc} \), and
- to every state \( \not\in \text{Pass} \cup \text{Inconc} \cup \text{Fail} \) without outgoing Exp-Trans (of type 1 or 3), we add a transition labelled \( \delta \) and leading to \( \text{inconc}_\delta \).

The use of \( \delta \) and \( \text{inconc}_\delta \) can be intuitively explained as follows: in a test execution if nothing happens during time \( T \), then the verdict Inconclusive is generated. For the \( \text{SpecTP}^{SE}_{\text{Vis}} \) of Fig. 8, we obtain the CTG of Fig. 9. In CTG, every input (resp. output) action must be interpreted as an output (resp. input) action of the tester (i.e., Test-Controller). Transition \( \delta \) in State 0 (resp. 1) is irrelevant, because it can be preempted by the only possible other transition labelled (\( ?\phi(m) ; - ; x := m \)) (resp. (\( ?\sigma, \text{Set}_{2,3} \))) which is under the control of Test-Controller. Transition \( \delta \) in State 4 indicates that nothing has happened during time \( T \), which implies the verdict Inconclusive. For simplicity, Fail and its incoming transitions are not represented; Fail is implicitly reached by every unspecified transition. Note that \( \delta \) can be easily implemented by using \( ?\text{Set}(c_0, T) \) and \( !\text{Exp}(c_0, T) \), where \( c_0 \) is a clock not used for describing timing constraints of Spec and TP.

Let \( \text{SpecTP}_A \) denote the part of \( \text{SpecTP} \) (obtained in Step 1) that leads to a location \( A \), and \( \text{SpecTP}^{SE}_{\text{Vis}}_A \) denote the part of \( \text{SpecTP}^{SE}_{\text{Vis}} \) (obtained in Step 2) that leads to a state \( A \). A verdict Pass, Inconclusive or Fail, generated after the execution by SUT of a trace \( \lambda \), is interpreted as follows:

- **Pass** means that \( \lambda \) conforms (w.r.t. \( \text{conf}^{SE}_{\text{Vis}} \)) to \( \text{SpecTP}^{SE}_{\text{Vis}}_A \). From Theor. 5.1, the IUT has executed a timed trace \( \mu \) that conforms (w.r.t. \( \text{conf}^{T}_{\text{Vis}} \)) to \( \text{SpecTP}_A \), and thus, to \( \text{SpecTP} \). Therefore, \( \mu \) conforms (w.r.t. \( \text{conf}^{T}_{\text{Vis}} \)) to Spec (because Spec and SpecTP are observationally equivalent).

- **Fail** means that \( \lambda \) does not conform (w.r.t. \( \text{conf}^{SE}_{\text{Vis}} \)) to \( \text{SpecTP}^{SE}_{\text{Vis}} \). From Theor. 5.1, the IUT has executed a timed trace \( \mu \) that does not conform (w.r.t. \( \text{conf}^{T}_{\text{Vis}} \)) to \( \text{SpecTP} \), and thus, nor to Spec (because Spec and SpecTP are observationally equivalent).

- **Inconclusive** means that we cannot determine whether \( \lambda \) conforms (w.r.t. \( \text{conf}^{SE}_{\text{Vis}} \)) to \( \text{SpecTP}^{SE}_{\text{Vis}}_A \). From Theor. 5.1, we cannot determine whether the IUT has executed a timed trace \( \mu \) that conforms (w.r.t. \( \text{conf}^{T}_{\text{Vis}} \)) to \( \text{SpecTP}_A \).

7. Contribution and future Work

Real-time test consists of testing systems which must guarantee timing constraints. Symbolic test consists of testing systems without enumerating values of their data. The contribution of this work is the combination of those two types of testing. An advantage of our method is its simplicity because the main treatment of the real-time aspect is concentrated into one step. Our method combines in a rigorous way the
method STG of symbolic testing of [Rusu et al., 2000] and the method of real-time testing of [Khoumsi et al., 2003]. Since the test method in [Khoumsi et al., 2003] is a rigorous generalization of TGV [Jard and Jéron, 2002] to the real-time case, we can say that our method is a rigorous generalization of STG and TGV\(^5\) to the real-time case. We are optimistic for the applicability of our method because both TGV and STG have led to interesting software tools. But we recognize that such applicability remains to be demonstrated with real world examples.

Theoretically, the method may suffer from state explosion essentially during the synchronized product (Step 1) and the transformation \(\text{SetExp}\) (Step 2). But in practice, the state explosion is attenuated by the following facts:

**For Step 1:** \(TP\) is relatively simple (see comment before Fig. 5).

**For Step 2:** the following two numbers, that influence state explosion, are relatively small:
- the number of clocks,
- the number of values to which each clock is compared in timing constraints.

Here are some future work directions:

- Our method (as well as STG in [Rusu et al., 2000]) does not support the quiescence aspect, that is used for specifying when the IUT is permitted to stop its execution. We intend to investigate the possibility to fill this gap.
- Our method (as well as other methods of real-time testing) does not support unobservable clock resets (Hyp. 5.1). We intend to determine conditions under which our method is applicable in the presence of unobservable clock reset.
- We intend to add the notion of forced actions, that is used to model actions that must occur (instead of being only permitted to occur) when they are enabled [Khoumsi et al., 2003].
- Def. 3.2 is not constructive and we do not know how to compute \(\text{InpComp}(S)\) from a \(T_{\text{in}} A\) in the general case. We intend to determine a class of \(T_{\text{in}}\)s for which we can construct this operator.
- We intend to implement a prototype of the test method in order to apply it and evaluate its complexity with non-trivial examples.

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**References**


\(^5\)Actually, STG and TGV are software tools for testing. But here, STG and TGV denote the theoretical test methods that underly the tools, respectively.


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