Generating Dynamically Stable Walking Patterns for Humanoid Robots Using Quadratic System Model

Wael Suleiman, Fumio Kanehiro, Kanako Miura and Eiichi Yoshida

Abstract—The approximation of humanoid robot by an inverted pendulum is one of the most used model to generate a stable walking motion. Many studies have been carried out to improve the reliability of this model. One of the proposed models is the quadratic system model, which has been validated by conducting real experiments on the humanoid robot.

In this paper, we propose several controlling algorithms for the quadratic system. Some of these algorithms are devoted for on-line and off-line walking pattern generation algorithms for the humanoid robot. Dynamically stable walking patterns have been generated in order to validate these algorithms.

The stability and feasibility of walking patterns have been confirmed using dynamical simulation and conducting real experiments on the humanoid robot HRP-4C.

Index Terms—Humanoid robot; ZMP control; Optimization; Nonlinear system control.

I. INTRODUCTION

Research on biped humanoid robot is nowadays one of the most active research fields in robotics. Making the humanoid robots walk was the subject of intense investigation last years. Many researchers have proposed different methods to generate a stable walking motion for humanoid robot [1]–[5]. Most of these methods are using a simplified model, which is based on the approximation of the humanoid robot by an inverted pendulum model. The mass of inverted pendulum coincide with the Centre of Mass (CoM) of the humanoid robot.

In order to generate a stable motion (dynamically balanced), these methods use the principle of Zero Moment Point (ZMP) [6]. The ZMP is a point of the support polygon (i.e. the convex hull of all points of contact between the support foot (feet) and the ground), at which the horizontal moments are vanishing.

The cart-table model proposed by Kajita et al [3] is one of these methods. The efficiency of this method has been proved by generating stable walking motions using a planned trajectory of ZMP. However, on account of the difference between this simplified model and the real dynamic of the humanoid robot, the generated walking patterns might be unstable if the error between those two models becomes bigger than the size of the support polygon [7]. One of the proposed models to improve the reliability of inverted pendulum model is the quadratic system model [7].

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Fig. 1: Walking pattern generation for the humanoid robot HRP-4C

The purpose in this paper is to give insights into the problem of controller design of quadratic system in order to generate dynamically stable walking patterns. The dynamical stability is guaranteed by tracking a pre-defined ZMP trajectory which is constrained to be always inside of the polygon of support. The proposed controlling algorithms are classified into two categories:

1) Off-line walking pattern generation algorithms: in this category the computational time is not considered, and the objective is to track the desired ZMP trajectory precisely.

2) On-line walking pattern generation algorithms: in this category the computational complexity of the controller should respect the real-time constraints. In this case, the desired ZMP trajectory is tracked as much as possible.

The paper is organized as follows. An overview of the cart-table model and the quartic system is discussed in Section II. The controller design problem, and the on-line and the off-line walking pattern generation algorithms are developed in Section III. The efficiency of the proposed walking pattern generation algorithms is shown in Section IV through dynamical simulation and conducting real experiment on the cybernetic human HRP-4C [8].
II. CART-TABLE MODEL AND QUADRATIC SYSTEM

The main idea of cart-table model [3] is to approximate the humanoid robot by a mass located at its Center of Mass (CoM) and it is equal to the total mass of the humanoid robot. Therefore, the complex problem of controlling the humanoid robot is transformed to controlling an inverted pendulum.

Let us define the Cartesian position of the center of mass ($P_{CoM}$) by

$$P_{CoM} = \begin{bmatrix} x \\ y \\ z_c \end{bmatrix}$$  \hspace{1cm} (1)

We suppose that the vertical position of the mass $z_c$ is constant.

The ZMP is a particular point of the horizontal plane at which the horizontal moments vanish. For the inverted pendulum, it is defined as follows

$$ZMP = \begin{bmatrix} p_x \\ p_y \end{bmatrix} = \begin{bmatrix} x - \frac{z_c}{g} \dot{z} \\ y - \frac{z_c}{g} \dot{y} \end{bmatrix}$$  \hspace{1cm} (2)

where $g$ is the absolute value of gravity acceleration.

It is clear from (2) that the two elements of ZMP are very similar. For that, in the sequel, we figure out how to build the control model for $p_x$.

To control $p_x$, Kajita et al [3] proposed to consider the cart-table model, which is shown in Figure 2. It depicts a running cart of mass $m$ on a pedestal table whose mass is negligible. In our case, this mass is equal to the total mass of the humanoid robot and its center coincides with the CoM of humanoid robot.

![Cart-table model](image)

Fig. 2: Cart-table model

A. Controlling ZMP

Let us define a new variable $u_x$ as the time derivative of $\dot{x}$

$$u_x = \frac{d\dot{x}}{dt}$$  \hspace{1cm} (3)

The variable $u_x$ is usually called the jerk.

Regarding $u_x$ as the input of $p_x$, we can translate the equation of $p_x$ into the following dynamical system

$$\frac{d}{dt} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_x$$

$$p_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix} x + \begin{bmatrix} -\frac{z_c}{g} \\ 0 \end{bmatrix} \dot{x}$$  \hspace{1cm} (4)

As the humanoid robot is controlled in discrete time, we discretize the system (4) with sampling time $T_s$. The control model for $p_x$ can be expressed by the following formula

$$z_k = A z_{k-1} + B \bar{u}_k$$

$$p_k = C z_k$$  \hspace{1cm} (5)

where

$$z_k \triangleq [x(kT_s) \hspace{0.2cm} \dot{x}(kT_s) \hspace{0.2cm} \ddot{x}(kT_s)]^T$$

$$\bar{u}_k \triangleq u_x(kT_s)$$

$$p_k \triangleq p_x(kT_s)$$

$$A = \begin{bmatrix} 1 & T_s & T_s^2 / 2 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix}, \hspace{0.5cm} B = \begin{bmatrix} T_s^3 / 6 \\ T_s^2 / 2 \\ T_s \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & -\frac{z_c}{g} \end{bmatrix}$$

Regarding the similarity of the equations of $p_x$ and $p_y$, the above equation can be extended to obtain the ZMP based control model as follows

$$X_k^1 = A_1 X_{k-1}^1 + B_1 u_k$$

$$p_k = C_1 X_k^1$$  \hspace{1cm} (6)

where

$$X_k^1 \triangleq \begin{bmatrix} x(kT_s) \\ \dot{x}(kT_s) \\ \ddot{x}(kT_s) \\ y(kT_s) \\ \dot{y}(kT_s) \\ \ddot{y}(kT_s) \end{bmatrix}$$

$$u_k \triangleq \begin{bmatrix} u_x(kT_s) \\ u_y(kT_s) \end{bmatrix}$$

$$p_k \triangleq \begin{bmatrix} p_x(kT_s) \\ p_y(kT_s) \end{bmatrix}$$

$$A_1 = I_2 \otimes A$$

$$B_1 = I_2 \otimes B$$

$$C_1 = I_2 \otimes C$$

$I_2$ is the identity matrix of dimension two and $\otimes$ is the operator of Kronecker product.

The above model has been used to generate a stable biped waking patterns using preview controller [3]. However, when the error between the output of this model and that of the real ZMP of humanoid robot becomes bigger than the stability margin the robot will fall down. The solution proposed by Kajita [3] is to inject this error in the preview controller as a second stage in order to eliminate the dynamic error. However, this procedure require the dynamic simulation of the multi-body model of the humanoid robot, as a result it is a time consuming process.
To overcome this problem, we proposed [7] to model the ZMP of the humanoid robot by the model given in Figure 3. This model consists of two blocks, the first one is the previous cart-table model and the second one is a linear system with respect to the input $u_k \otimes X_k^1$. The main objective of the second block is to capture the dynamic behavior that is related to the difference between the simple cart-table model and the multi-body model of the humanoid robot.

![Cart-Table Model](image)

**Fig. 3: Proposed ZMP model**

The obtained model is a quadratic system in the state space representation. This realization of quartic system is given by the following structure

$$
\begin{align*}
X_k^1 &= A_1 X_{k-1}^1 + B_1 u_k \\
X_k^2 &= A_2 X_{k-1}^2 + B_2 (u_k \otimes X_k^1) \\
p_k &= C_1 X_k^1 + C_2 X_k^2
\end{align*}
$$  

where $u_k \in \mathbb{R}^2$ is the input signal of the system expressed as in Eq. (8). $p_k \in \mathbb{R}^2$ is the output of the system (ZMP).

$\{X_k^i : i = 1, 2\}$ are the states, the dimension of $X_k^1$ is $n_1$ (in our case $n_1 = 6$), and the dimension of $X_k^2$ is $n_2$ that will be determined by the identification algorithm.

It has been proven [9], [10] that this class of nonlinear system is a special case of Volterra series of degree two. In practice, this class can play a useful role to model many real systems.

The direct relation between $p_k$ and $u_k$ can be obtained by using the property $FG \otimes HJ = (F \otimes H)(G \otimes J)$ and a simple development of (9)

$$
p_k = \sum_{i=1}^{k} \Phi^1(k, i) u_i + \sum_{i=1}^{k} \sum_{j=1}^{i} \Phi^2(k, i, j) (u_i \otimes u_j)
$$  

where

$$
\begin{align*}
\Phi^1(k, i) &= C_1 (A_1)^{k-i} B_1 \\
\Phi^2(k, i, j) &= C_2 (A_2)^{k-i} B_2 \left[ I_2 \otimes ((A_1)^{i-j} B_1) \right]
\end{align*}
$$  

III. CONTROLLER DESIGN

In this paper we focus on the problem of controller design. The problem of designing a controller to follow a desired ZMP trajectory can be formulated as follows

$$
\begin{align*}
\min_{u_k} \sum_{k=0}^{L} u_k^T Q u_k + (p_k^{ref} - p_k)^T Q_e (p_k^{ref} - p_k) \\
\text{subject to} \quad X_k^1 &= A_1 X_{k-1}^1 + B_1 u_k \\
X_k^2 &= A_2 X_{k-1}^2 + B_2 (u_k \otimes X_k^1) \\
p_k &= C_1 X_k^1 + C_2 X_k^2
\end{align*}
$$  

where $p_k^{ref}$ designs the desired ZMP trajectory. $L$ is the number of last sampling of the trajectory.

In this section we propose one algorithm for off-line and on-line walking pattern generation algorithms based on controlling the quadratic system. The off-line algorithm is useful, for instance, in the case of motion planning. In this case, the path of the humanoid robot is provided off-line. However, for the reactive motions and human-robot interactions the on-line controlling algorithms are crucial.

A. Off-line walking pattern generation Algorithm

It is well known that the space of the admissible solutions of the minimization problem (11) is very large.

In order to transform this space to a smaller dimensional space, we can use a basis of shape functions.

Let us consider a basis of shape functions $B_k$ that is defined as follows

$$
B_k = \begin{bmatrix} B_k^1 & B_k^2 & \cdots & B_k^{l} \end{bmatrix}^T
$$  

where $B_k^i$ denotes the value of shape function number $i$ at the instant $k$. The dimension of $B_k$ is $l$ which defines the dimension of the basis of shape functions.

The projection of $u_k$ into the basis of shape functions $B_k$ can be given by the following formula

$$
u_k = (I_2 \otimes B_k)^T u_B
$$  

where $I_2 \in \mathbb{R}^{2 \times 2}$ denotes the identity matrix, and $u_B \in \mathbb{R}^{2l}$ denotes the vector of control points of the B-spline functions.

Thus, the optimization problem (11) can be rewritten as follows

$$
\begin{align*}
\min_{u_B} \sum_{k=0}^{L} u_k^T (I_2 \otimes B_k) Q u_k (I_2 \otimes B_k)^T u_B + \cdots
\end{align*}
$$  

where

$$
\begin{align*}
\min_{u_B} \sum_{k=0}^{L} u_k^T (I_2 \otimes B_k) Q u_k (I_2 \otimes B_k)^T u_B + \cdots
\end{align*}
$$  

By defining the following variables

$$
\begin{align*}
\min_{u_B} \sum_{k=0}^{L} u_k^T (I_2 \otimes B_k) Q u_k (I_2 \otimes B_k)^T u_B + \cdots
\end{align*}
$$  

subject to

$$
\begin{align*}
X_k^1 &= A_1 X_{k-1}^1 + B_1 (u_k \otimes X_k^1) \\
X_k^2 &= A_2 X_{k-1}^2 + B_2 (u_k \otimes X_k^1) \\
p_k &= C_1 X_k^1 + C_2 X_k^2
\end{align*}
$$  

where $p_k^{ref}$ designs the desired ZMP trajectory. $L$ is the number of last sampling of the trajectory.
\[ J = \sum_{k=0}^{L} u_k^T (I_2 \otimes B_k) Q_u (I_2 \otimes B_k)^T u_B + \cdots (p_k^{ref} - p_k)^T Q_e (p_k^{ref} - p_k) \]

\[ H = \begin{bmatrix}
    X_1^k - A_1 X_1^{k-1} - B_1 (I_2 \otimes B_k)^T u_B \\
    X_2^k - A_2 X_2^{k-1} - B_2 \left( (I_2 \otimes B_k)^T u_B \right) \otimes X_1^k \\
    p_k - C_1 X_1^k - C_2 X_2^k
\end{bmatrix} \]

The optimization problem (14) is transformed into the following classical optimization problem

\[
\begin{aligned}
\min_{u_B} & \quad J(u_B) \\
\text{subject to} & \quad H(u_B) = 0
\end{aligned}
\]  

(15)

In other words, the optimization problem has been transformed into finding the vector \( u_B \in \mathbb{R}^{2n} \).

Note that the trajectory of CoM of the robot can be directly obtained from \( X_1^k \) by applying the input signal \( u_k \) to the quadratic system (9).

B. On-line Walking Pattern Generation Algorithm I

The idea of this algorithm is linearizing the quadratic system, then controlling the linearized system.

The linearized system of quadratic system (9) around the value \( X_1^k = X_1^* \) is given by

\[
\begin{aligned}
    X_1^k &= A_1 X_1^{k-1} + B_1 u_k \\
    X_2^k &= A_2 X_2^{k-1} + B_2 (u_k \otimes X_1^*) \\
    p_k &= C_1 X_1^k + C_2 X_2^k
\end{aligned}
\]  

(16)

Let us define the matrix \( B_{2,j} \) as follows

\[ B_{2,j} = B_2(:,(j-1)n_1 + 1:jn_1) \]  

(17)

where \( n_1 \) denotes the dimension of \( X_1^k \) (in our case \( n_1 = 6 \)), and the notation \( M(:,i:j) \) designs the sub-matrix of matrix \( M \) which contains the columns from \( i^{th} \) to \( j^{th} \) column. Using (17) the quantity \( B_2(u_k \otimes X_1^*) \) can be formulated as follows

\[
B_2(u_k \otimes X_1^*) = \begin{bmatrix}
    B_{2,1} X_1^k & B_{2,2} X_1^k & \cdots & B_{2,m} X_1^k
\end{bmatrix} u_k
\]

\[ \triangleq B_2^* u_k \]  

(18)

Recall that \( u_k \in \mathbb{R}^2 \).

Using the above definitions, the linearized system (16) can be reformulated as follows

\[
\begin{aligned}
    X_k &= A X_{k-1} + B u_k \\
    p_k &= C X_k
\end{aligned}
\]  

(19)

where

\[
\begin{aligned}
    X_k &= \begin{bmatrix} X_1^k \\ X_2^k \end{bmatrix} \\
    A &= \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2^* \end{bmatrix} \\
    C &= \begin{bmatrix} C_1 & C_2 \end{bmatrix}
\end{aligned}
\]  

(20)

Therefore, the controller designing problem (11) is transformed into the following problem

\[
\begin{aligned}
\min_{u_k} & \quad \sum_{k=0}^{L} u_k^T Q_u u_k + (p_k^{ref} - p_k)^T Q_e (p_k^{ref} - p_k) \\
\text{subject to} & \quad X_k = A X_{k-1} + B u_k \\
& \quad p_k = C X_k
\end{aligned}
\]  

(21)

The above optimization problem is well known in automatic control as the preview control of a linear system. The optimal input signal \( u_k \) can be obtained by applying the classical Linear Quadratic Gaussian (LQG) technique. Thus, when the ZMP reference is previewed for \( N_L \) future steps at every sampling time, the optimal controller which minimizes the objective function (21) is given by

\[
\begin{bmatrix}
    p_{k+1}^{ref} \\
    \vdots \\
    p_{k+N_L}^{ref}
\end{bmatrix}
\]

(22)

where \( K \) and \( f_i \) are calculated as

\[
K = \left( R + B^T P B \right)^{-1} B^T P A
\]

\[
f_i = \left( R + B^T P B \right)^{-1} B^T \left( A^T - K T B^T \right)^{(i-1)} C^T Q
\]  

(23)

\( P \) is the solution of the following Riccati equation

\[
P = A^T P A + C^T Q C - A^T P B \left( R + B^T P B \right)^{-1} B^T P A
\]  

(24)

Note that the linearized system (16) is valid for a region around the value \( X_1^k = X_1^* \). Therefore a threshold \( \epsilon > 0 \) should be defined and the linearized system is updated with the new value of \( X_1^k \) if \( \| X_1^k - X_1^* \| > \epsilon \).

The trajectory of CoM can be obtained directly from \( X_1^k \) by simulating the quadratic system (9) using the control signal \( u_k \).

C. On-line Walking Pattern Generation Algorithm II

The key idea of this algorithm can be summarized as follows

1) Controlling the quadratic system as a linear system.
   The approximated linear system is the linear subsystem (Cart-table model) of the quadratic system.
2) Calculating the error between the dynamic simulation of the linear system and the quadratic system
3) Correcting the error by considering it as perturbation, and calculate the appropriate input signal to decrease its effect.
The application of the above steps leads to at first solving the following problem

$$\min_{u_k} \sum_{k=0}^{L} u_k^T Q_u u_k + (p_k^{ref} - p_k)^T Q_e (p_k^{ref} - p_k)$$

subject to

$$\dot{X}^1_k = A_1 \dot{X}^1_{k-1} + B_1 u_k$$
$$p_k = C_1 \dot{X}^1_k$$

(25)

Analogously to the previous controlling algorithm the above optimization problem can be solved using LQG technique.

The second step is to find the error between the outputs of the linear subsystem and the quadratic system by simulating the systems (25), (9) using the obtained input signal $u_k$, and obtain

$$\Delta p_k^{ref} = C_1 X^1_k + C_2 X^2_k - C_1 \dot{X}^1_k$$

(26)

In order to decrease the effects of this error one can consider it as an external perturbation, and apply LQG once more to calculate the appropriate variation of the input signal $u_k$.

Thus, the new LQG problem can be formulated as follows

$$\min_{\Delta u_k} \sum_{k=0}^{L} \Delta u_k^T Q_u \Delta u_k + (\Delta p_k^{ref} - \Delta p_k)^T Q_e (\Delta p_k^{ref} - \Delta p_k)$$

subject to

$$\dot{X}^1_k = A_1 \dot{X}^1_{k-1} + B_1 \Delta u_k$$
$$\Delta p_k = C_1 \Delta \dot{X}^1_k$$

(27)

Analogously to (25) the variation of the input signal $\Delta u_k$ can be obtained. As a result the input signal that should be applied becomes $\tilde{u}_k = u_k + \Delta u_k$.

IV. EXPERIMENTAL RESULTS

A. Humanoid Robot: Kinematic Structure

The proposed walking patterns generation algorithms have been validated using the cybernetic human HRP-4C [8]. HRP-4C is a life-size humanoid robot which was developed for the entertainment use such as a fashion model, master of ceremony of various events and so on. For these applications, it has been decided to make it more humanlike than the humanoid robots that have developed so far [11], [12]. In order to realize a humanlike shape, its dimensions are designed referring to a database of Japanese women of 20 century [13]. The basic specifications and joint configurations are respectively shown in Table I and Figure 4(right) (Face and hand joints are not displayed).

By applying the method proposed in [7], we can identify the quadratic system related to the walking motions of the cybernetic human HRP4-C. In order to validate the three controlling algorithms proposed in this paper, we generate walking patterns to follow a desired ZMP trajectory. Note that the controlling algorithms provide as output the trajectory of CoM, then the whole body motion of the humanoid robot is calculated using inverse kinematics in order to follow the desired trajectory of CoM.

B. Results: Off-line Walking Pattern Generation Algorithm

The application of the off-line walking pattern generation algorithm to generate the CoM trajectory in order to track a desired trajectory of ZMP is applied to the humanoid robot HRP4-C. We have chosen a basis of 40 B-spline functions. The optimization algorithm stops when the norm of error between the planned ZMP ($p^{ref}$) and the output of quadratic system is less than 10$^{-4}$. The planned ZMP trajectory and that of the multi-body model are given in Figure 5, the error between those two trajectories is given in Figure 6.

Note that the reported error in Figure 6 and the ZMP trajectory in Figure 5 are those of multi-body model of the humanoid robot, and they are not those of quadratic system.

From Figures 5 and 6, we can conclude the following remarks

1) The planned ZMP trajectory is well tracked by the proposed controller.

2) The error between the ZMP of multi-body model of humanoid robot and that of the desired ZMP trajectory ($p^{ref}$) is small enough (less than 2 cm), and the ZMP stays inside of the polygon of support. Moreover, that proves that the quadratic system is a reliable model of the walking motion.

In order to validate the obtained results, we have simulated the generated motion using the dynamic simulator.

![Fig. 4: Exterior and joint configuration of Humanoid Robot HRP-4C (Face and hand joints are not displayed)](image)

### TABLE I: Specifications of HRP-4C

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Fig. 5: Off-line walking pattern generation algorithm: the desired ZMP trajectory and that of multi-body model of humanoid robot.

Fig. 6: Off-line walking pattern generation algorithm: the ZMP error between the multi-body model of humanoid robot and the desired ZMP trajectory.

Fig. 7: Off-line walking pattern generation algorithm: snapshot of the simulated walking motion.

Fig. 8: On-line walking pattern generation algorithm I: the ZMP error between the multi-body model of humanoid robot and the quadratic system.

C. Results: on-line Walking Pattern Generation Algorithm I

In this experiment, we applied the on-line walking pattern generation algorithm I to generate the CoM trajectory of the humanoid robot. The error between those two trajectory is given in Figure 8.

From Figure 8, we observe that the ZMP trajectory is always inside of the polygon of support. However, the ZMP tracking error is bigger than that of off-line walking pattern generation. On the other hand, this controller can be applied on-line on account of its low computational complexity.

Snapshots of the real experiment conducted on the humanoid robot HRP-4C are given in Figure 9.

D. Results: on-line Walking Pattern Generation Algorithm II

Analogously to the previous case, we applied the on-line walking pattern generation algorithm II to generate the CoM trajectory. The error between those two trajectory is given in Figure 10.

From Figure 10, we observe that the ZMP tracking error becomes bigger. However, the ZMP is always inside of the polygon of support, and the dynamical stability of the walking patterns has been verified using the dynamical simulator OpenHRP3.

V. Conclusion

The quadratic system [7] has been proven to be more reliable model for humanoid robot walking motion than the simple inverted pendulum model. In this paper, the problem of controller design of quadratic system has been...
investigated. We have proposed on-line and off-line walking pattern generation algorithms for the humanoid robots based on controlling the quadratic system. Even though the off-line walking pattern generation algorithm can track the pre-defined ZMP trajectory precisely, the computational time is not adequate for real-time applications. On the other hand, the on-line walking pattern generation algorithms are devoted for real-time applications and the ZMP trajectory is tracked as much as possible. The conducted simulations using the dynamical simulator OpenHRP3 [14], [15] and the real experiments using the cybernetic human HRP4-C [8] have pointed out that the ZMP trajectory stays always inside of the polygon of support, and the generated walking patterns are dynamically stable.

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