A Spiking Neural Network For
Spatio-Temporal Pattern Detection

Tuong Vinh HO1,2  Jean ROUAT1
1Université du Québec à Chicoutimi, Canada, Dept. des Sciences Appliquées
2Ecole Polytechnique de Montréal, Canada, Dept. de Génie Informatique
email: vho@uqac.uquebec.ca  Jean_Rouat@uqac.uquebec.ca

Abstract

We propose a spiking neural network model that is inspired from an oversimplified general
structure of layer IV of the cortex. The neuron model is an integrate-and-fire model whose
threshold changes depending on its firing activity. Firing activity of neurons in combination with
interactivity between them creates a highly dynamical self-organized process. The spiking
activity, the neuron’s threshold that changes depending on the firing activity and the time of
stabilization of the proposed network can be used to represent its complex behavior. We
experimentally show that this representation of the complex behavior of the network can be used
to characterize spatio-temporal information of input patterns. Also, a new paradigm for novelty
detection by the spiking neural network is proposed. The stabilization time of the neural network
is used as a criterion for novelty detection.

Through a comparison study with other networks as Hopfield, backpropagation and DYSTAL
network, we show that the proposed network can yield a comparable performance with that of a
backpropagation network and a Hopfield network for a noisy digit recognition task. Furthermore,
grouping and creation of sets of synchronized neurons has been observed. The sets depend on the
input patterns. This suggests that patterns detection and recognition could be performed based on
the grouping and on the synchronization of the spiking neurons. This work is a possible bridge between nonlinear dynamical systems and neural networks applications in spatio-temporal pattern recognition.

Keywords: Integrate-and-Fire neurons, Spiking neural networks, Spatio-temporal patterns, Self-organization, Cortical networks, Synchronized neurons.

1 INTRODUCTION

The processing or the recognition of non-stationary noisy process with Neural Networks is a challenging and yet unsolved issue. Most of the contemporary pattern processing or recognition techniques assume that the pattern or the time-varying signal to be recognized is stationary. Furthermore, in real life applications, the information is most of the time corrupted, partial or noisy (image, speech, etc.). Therefore, the pattern recognizers have also to be robust.

Among the many neural network systems that are reported in the literature, we can find neural networks with complex behavior or oscillatory dynamics. We are interested in evaluating such neural networks as potential recognizer systems of non-stationary noisy processes. In the present work we propose a spiking neural network and a first evaluation by comparing with standard networks on a novelty detection task of noisy digits.

Representation of information in the nervous system has often been considered as being contained in simultaneous discharges of a large set of neurons. How does a neural system use that kind of information representation while performing learning and pattern recognition? Recent studies on nonlinear cooperative complex dynamics in neural systems provide various kinds of models that describe the cooperative behavior such as synchronization and chaos (e.g., [5], [6],
Especially, a valuable overview on some principles for information storage and retrieval based on oscillations in dynamical systems is given in [25].

In the field of modeling information processing in biological neural systems, many works are focused on networks of spiking neurons. Experimental evidence has accumulated during the last years which indicates that many biological neural systems use the timing of single action potentials («spike») to encode information ([12], [21], [22]). The timing of individual potential activity plays a key-role for computations in networks of spiking neurons. It is demonstrated in [20] that networks of spiking neurons are computationally more powerful than other neural network models based on McCulloch Pitts neuron (i.e. threshold gates). Another work [19] suggests the use of action potential timings to encode analogue information and propose time-delay networks to process this representation. Such a coding scheme has a great computational power and a better speed in comparison with networks using an «instantaneous rate» coding of variables.

Synfire chains have been proposed as a mechanism for neural information processing in the cortex [1]. The formation of synfire chains through a biologically plausible self-organizing mechanism is investigated in [15]. A network model of cortical neurons capable of learning synfire chains by introducing a Hebbian learning mechanism is proposed. However, this type of network is unstable against the formation of long synfire chains. Other works ([16]) developed neural models to study the spatio-temporal pattern generation properties in a simulated «cortical neural network». The model uses integrate-and-fire neurons as elementary units. Furthermore, the topology is inspired from that of layer IV in the cortex. Although this model helps to observe the evolution of spatio-temporally organized activity in a simulated cortex, the learning rule is not proposed.
We propose an unsupervised spiking neural network model that is inspired from the simulated cortical neural network. The model uses a global controller to regulate the firing activity of a network of spiking neurons. The system dynamics and the self-organizing process are complex and difficult to characterize. The dynamics are sometimes periodic or non-periodic. One of the objectives of the present work is to explore the usefulness of the network to perform novelty and pattern recognition tasks without any *a priori* knowledge of the exact dynamics and characterization of the mathematical properties of the network. In fact, it is difficult to find the relation between the stimulus input and the convergence of the network towards specific "attractors". Furthermore, there is no universal agreement in the scientific community on the techniques that can be used to characterize such properties. Therefore, we propose the use of the stabilization time of the complex system as a criterion for novelty detection. When input is similar to one of learned patterns, the network reaches an equilibrium state very quickly. In contrast, when input is not too similar from any learned patterns, the time of stabilization of the network is longer. In addition, a performance comparison with other networks such as the Hopfield, backpropagation and DYSTAL networks for a noisy character recognition task is presented.

The paper is organized as follows: Section 2 describes the neuronal model, architecture and learning mechanism of the network model. Empirical studies of the self-organizing activity of the network model are given in section 3. Section 4 introduces the novelty detection paradigm based on the stabilization time of the network. The present work is discussed and future directions are suggested in section 5.

2 NEURAL NETWORK MODEL
2.1 Neuronal Model

The neuron model is inspired from the integrate-and-fire neuronal model proposed in [16] with refractory period and post-synaptic potential decay. The state of the neuron \( S_i \) at time \( t \), is deterministically modeled by a control potential, \( U_i(t) \) as:

\[
S_i(t) = \begin{cases} 0 & \text{if } (t - t_{\text{spike}}) < \rho, \\ H[U_i(t) - \theta] & \text{otherwise}, \end{cases}
\]

where \( \theta \) is the threshold and \( H \) is the Heaviside function defined as \( H[x] = 1 \) for \( x > 0 \) otherwise \( H[x] = 0 \). The value \( t_{\text{spike}} \) represents the last firing time for unit \( i \), \( \rho \) is the absolute refractory period. \( \rho \) is the period during which the cellular biochemical mechanisms cannot generate another signal regardless of the strength of the stimulation.

The control potential \( U_i(t) \) is defined as:

\[
U_i(t) = U_{i,\text{int}} + U_{i,\text{ext}}(t)
\]

where \( U_{\text{int}} \) is the resting potential and is constant, \( U_{i,\text{ext}}(t) \) is the external potential. The external potential \( U_{i,\text{ext}}(t+1) \) is defined as the integration of all afferent postsynaptic potentials at time \( t \):

\[
U_{i,\text{ext}}(t+1) = U_{i,\text{ext}}(t) + \sum_j C_{ij}(t)S_j(t) + \delta U_{i,\text{ext}}(t) + e_i(t)
\]

where the indices \( i \) and \( j \) characterize the neurons, \( C_{ij}(t) \) is the connection strength, and \( e_i(t) \) is the external input signal. The external potential decays following the first order kinetics \( \kappa \) with \( \delta = \exp(-\ln(2)/\kappa) \). It is also reset to the value \( \text{rest}(0) \) after a spike.

In order to take into consideration the influence of the firing activity into the neuron’s behavior, we introduce a variable threshold in equation (1). The threshold \( \theta(t) \) is defined as

\[
\theta(t) = b_i(t) - a_i(t)
\]
where $a_i(t)$ and $b_i(t)$ are factors that respectively correspond to firing and non-firing activity of the neuron. At each instant, if the neuron fires the firing factor $a_i(t)$ is increased. Otherwise, the non-firing factor $b_i(t)$ is increased. In other words, if the neuron fires its threshold will decrease (Equation 4). This facilitates the neuron firing activity. In contrast, if the neuron does not fire, its threshold is increased, therefore it becomes less excitable. Therefore, the neuron’s threshold is quite dependent on its firing activity. These $a_i(t)$ and $b_i(t)$ factors are updated during simulation course as follows:

$$a_i(t + 1) = a_i(t) + \lambda a_i(t)(1 - a_i(t))$$  \hspace{1cm} (5) \\
$$b_i(t + 1) = b_i(t) + \lambda b_i(t)(1 - b_i(t))$$  \hspace{1cm} (6)

where $\lambda$ is the update rate, $a_i(t)$ and $b_i(t)$ are initialized by a positive fixed value smaller than 1.

From equations 5 and 6 one can realize that the factors $a_i(t)$ and $b_i(t)$ evolve in time according to a logistic function. The amount of change for these factors is proportional to their derivative. The expression $a(t)(1-a(t))$ is the derivative of the logistic function $a(t) = 1 / (1+\exp(-t))$. It is important to note that these factors are updated only when the neuron is not in its refractory period.

2.2 Network Architecture

The network architecture is inspired from an oversimplified model of cortical layer IV ([16]). This model defines a single two-dimensional sheet of excitatory and inhibitory neurons with recurrent connections. The layer consists of two populations of neurons interspersed within the plane. These neurons are positioned according to a space-filling pseudo-random Sobol distribution. In our model, each neuron has a set of interconnections chosen according to a square neighborhood, centered on the neuron itself. Excitatory and inhibitory neurons can have different
neighboring radius. In fact, the neighboring radius is a variable parameter. From this topology, we can say that the model has an interactivity at local level between neighboring neurons. In order to take into account interactivity at global level (between all neurons in the network) we introduce a global controller (Figure 1). With this approach, a self-organizing evolution arisen inside the network will be a result of interactivity at both levels: local and global.

The global controller is actually a trigger whose state is either active, i.e. firing, or inactive depending on a control mechanism. The control mechanism is based on a threshold for the total number of firing neurons. Whenever the number of firing neurons at time $t$ is above the threshold, the global controller fires and generates a negative feedback signal (-1) to every neuron in the network. Otherwise, if the number of firing neurons is below the threshold, the global controller generates a positive feedback signal (+1). Thus, the global controller regulates the network activity at a global level. With a feedback signal $h$ ($h = -1$ or $+1$), equation (3) becomes:

$$U_{i, ext}(t + 1) = \sum_j C_{ij}(t)S_j(t) + \delta U_{i, ext}(t) + \epsilon_i(t) + G_i(t)h(t)$$  \hspace{1cm} (7)

$G_i(t)$ is the connection strength between the global controller and neuron $i$.

Figure 1. The network architecture: Samples of neighboring connections of excitatory (a) and inhibitory (b) neurons with radius equal 2 and 1 respectively.

2.3 Learning Mechanism

In the proposed model, there are two kinds of coupling: one is between neighboring neurons and the other is between the global controller and all neurons. The coupling weights between a pair of neurons are updated according to a rule proposed in [23]:

$$C_{ij}(t + 1) = C_{ij}(t) + \alpha C_{ij}(t)(1 - C_{ij}(t))S_i(t)S_j(t) \sum_j C_{ij}(t)$$  \hspace{1cm} (8)
where $\alpha$ is the learning rate.

The coupling weights between each neuron and the global controller are updated according to:

$$G_i(t + 1) = G_i(t) + \beta G_i(t)(1 - G_i(t))S_i(t)h(t)$$  \hspace{1cm} (9)

where $G_i(t)$ is the coupling weight of the neuron $i$, $\beta$ is the update rate and $h(t)$ is the feedback signal from the global controller.

The coupling weights of the network $(C_{ij}(t), G_i(t))$ are updated as soon as the network is stimulated by an input signal. At each instant, the coupling weights of a neuron are updated if this neuron fires. It also receives a feedback signal from the global controller. The later is either negative (-1) or positive (+1) depending on the number of firing neurons above or below the threshold of firing neurons. The network is considered to reach a stable state when its local coupling weights $C_{ij}(t)$ do not change anymore or when they change in a very small given range (example 0.001). There is no difference between "training" and recognition (or novelty detection), the "training" is always performed except when the network is considered to be stable.

### 3 SELF-ORGANIZING ACTIVITY

In this section, we study the behavior and dynamic characteristics of the network model in two following situations: when it is stimulated by only one stimulus and when it is stimulated by a sequence of stimuli.

#### 3.1 Simulation Conditions

A set of 0-9 digits was used as a pattern set. Each digit pattern was coded by using a 7x5 binary pixel matrix (Fig. 2). In order to test the robustness of the network, a set of noisy patterns
obtained from the original patterns (Fig. 2, columns 1 and 4) was also used. The noisy patterns were created by adding noise to the original pattern images. In other words, given an amount of noise (by percentage), a fix number of pixels in the patterns were changed. The pixels to be changed were randomly chosen with a uniform probability distribution. In this experiment, with a 20% of noise, 7 pixels in each 7x5 pixel image had their value changed. Though the pattern images used herein were binary images, the network can manipulate analog images (i.e. real numbers can be manipulated by the network).

**Figure 2.** Patterns of 0-9 digits (columns 1 and 4: clean digits; columns 2, 3 and 5, 6 with 20% noise).

We used a 7x5 dimension network with 70% of the population being excitatory neurons and the remaining 30% being inhibitory neurons. The neighborhood radius was 2 for excitatory neurons and 1 for inhibitory neurons. The neurons are numbered (1 to 35) from left to right and from top to bottom in the network plane. The pattern image was positioned at the center of the network plane. Thus, every \( i \) neuron covered by the pattern image received an input signal \( e_i(t) \) from its corresponding pattern image pixel. Uncovered neurons received an input signal equal to 0. The network’s parameters were initialized as \( a(0) = 0.1, b(0) = 0.1, U_{int} = -3.0, \kappa = 3.0, \rho = 3.0, \alpha = 0.2, \beta = 0.2, \lambda = 0.1, \) and the threshold for the number of firing neurons was 50% of the total neurons. The connection weights were initialized randomly in a range from 0 to 1. Time was discrete and measured in 1-iteration units.

### 3.2 Self-organization with one input pattern

#### 3.2.1 Stabilizing activity
The state of the network during the self-organizing phase cannot be analyzed by means of a simple algorithm. When patterns are introduced into the network, it is activated and it shows a complex behavior. Then, the network reaches a stable state where the connection strengths change in a very small range. In order to better understand the dynamical behavior of the network, we studied the evolution of the connection weights during the learning phase. For simplicity, the maximum value of the change in connection weights at each instant is used to characterize the stabilization. That value is defined as the maximum weight difference \( m \):

\[
m = \max \{ C_i(t + 1) - C_i(t) \} \text{ for all } i, j.
\]  

(10)

Hereafter, we use the maximum weight difference \( m \) as a representation of the dynamical evolution of the network. We will refer to \( m \) as being the maximum weight distance.

The evolution of the weight distance (\( m \)) of the network after being stimulated by the digits 0 and 2 respectively is not similar (Figure 3). The same initial conditions are used when presenting digit 0 or 2. We can identify a learning period during which the maximum weight distance is high and pseudo-oscillating. After an interval \( T \), that distance does not change any more or in a very small range. Thus, we consider that the network reaches a stable state and that the learning or recognition were completed. When performing the recognition, the network should provide the same response pattern for several similar input patterns.

Figure 3. Maximum weight difference \( m \) evolution when one input pattern is presented: a) digit 0 and b) digit 2. The same initial conditions are used before presenting the stimulus.

In order to observe the organizing behavior when a pattern is presented many times to the network, we introduced the digit 0 five times to the network. The network reaches a stable state after 660 iterations for the first presentation of 0. Then, after each subsequent presentation, a
stable state was observed after 11 iterations. The same experiment was reproduced with digit 2 where the network reaches a stable state after 768 iterations for the first presentation, and then 11 iterations for the subsequent presentations. Therefore, the network seems to have a good ability of adaptation by self-organization.

3.2.2 Firing activity

For spiking neural networks, the firing activity of all neurons can be considered as a spatio-temporal pattern with an explicit temporal structure. Also, for the proposed network, its spiking activity can be observed through the firing phase of the neurons, the firing rate and the threshold. The set of neurons covered by the digit 0 or 2 has a synchronous firing activity (Fig. 4a, b). The spiking neurons have the same firing phase and firing rate. The threshold of these active neurons is low ($\theta = -0.9$). The other neurons remain in silence with a high threshold $\theta = 0.9$ (Figure 5a, c).

Figure 4. Spiking activity of the network after being simulated by: a) digit 0 and b) digit 2 respectively.

Figure 5. Thresholds of the network after being stimulated by: a) digit 0 only; b) digit 0 and 2 sequentially; c) digit 2 only; d) digit 2 and 0 sequentially. Each value corresponds to the threshold of each neuron after stabilization of the network.

These observations suggest that the cells that fire synchronously with a lower threshold $\theta$ belong to the same set. That set of cells can characterize the input patterns. In other words the cells that do belong to the same set (i.e. "object") fire collectively and synchronously. In some sense, one could refer to synfire chains formed through a biologically plausible self-organizing mechanism. Also from this experiment, one can see that the network has different final spiking
patterns when it is stimulated by different input patterns. Depending on the input stimulus, the self-adaptation mechanism will generate different sets of synchronous cells. Another plausible interpretation is that the network has different final attractors whose nature depends on the input patterns. Thus, the network might be a multiple attractor system in the sense of non-linear systems. The self-adaptation mechanism "facilitates" the apparition of specific attractors depending on the input stimulus. We postulate that the spatio-temporal distribution of the synchronous cells with a low threshold characterizes the input patterns.

3.3 Self-Organization with a Set of Input Patterns

In the previous experiments, only a single pattern was used to stimulate the network and the connection strengths were randomly initialized. However, we need to understand the behavior of the network when a set of different patterns is sequentially presented as input. There will be no weight initialization between successive presentations of input patterns.

3.3.1 Experiment 1: Self-organization into sets of firing neurons

First, the network was stimulated by digit 0. It oscillated and reached a stable state after 660 iterations. Then digit 2 was presented to the network that oscillated and reached another stable state after 748 iterations. The threshold of all neurons was reset to the same initial value before presentation of the pattern digit 2 to the network. The threshold of all neurons at final state is given in Figure 5b. Figure 6a shows the spiking activity during the transition phase from 0 to 2 (60 iterations before and after presenting digit 2).

Figure 6. Spiking activity of the network after being stimulated by: a) digit 0 and 2 sequentially; b) digit 2 and 0 sequentially.
Neurons that are not covered by digits 0 or 2 remained silently (with $\theta = 0.9000$). Neurons covered only by digit 0 synchronized with a firing time lag of one time unit (set 1, neurons: 7, 12, 20, 22, 25, 27 and 30, with $\theta = -0.8910$). Neurons that were covered simultaneously by digits 0 and 2 also synchronized and the firing phase is not affected by the presentation of digit 2 (set 2, neurons: 3, 4, 10, 15, 17, 33 and 34, with $\theta = -0.9000$). It is likely that the learned pattern (digit 0) is still stored in the network and is not destroyed by the presentation of a new pattern. Neurons that are covered only by digit 2 can be separated into two sets (set 3 and set 4) based on their firing phase. Neurons belonging to set 3 (neurons 19, 31 and 32 with $\theta = -0.8708$), have the same firing phase as those of set 2 while the neurons from set 4 (neurons 2, 6, 18, 21, 26, 35, $\theta = -0.8813$), have a firing phase different from all other sets. It is likely that the synchronization of spiking activity into sets is a result of the continuous change of the threshold $\theta$ during simulation time.

In a second experiment, we used the sequence of digits 2 and 0 to stimulate the network with the same initial conditions. In contrast with the previous experiment, digit 2 was presented first and then digit 0. The stabilization times corresponding to digits 2 (768 iterations) and 0 (572 iterations) were different from those of the previous experiment (660 iterations for digit 0 and 748 iterations for digit 2). The network’s threshold and spiking activity are given in Fig. 5d and 6b respectively.

The set of neurons that are common to digit 0 and 2 (neurons with the same input $e_i(t)$ for digit 0 or 2) is unchanged and the firing activity does not seem to depend on the sequence presentation (set 2 neurons). The set of neurons that was labeled set 1 (input $e_i(t) = 1$ for digit 0) splits into two sets: The main set comprises neurons 7, 22, 25, 27 and 30 and the secondary set
comprises neurons 12 and 20. Sets 4 and 3 (input $e_i(t)=1$ for digit 2) merge also into a main set (2, 6, 18, 19, 21, 26 and 32) and a secondary set (31 and 35). In fact, the presentation of digit 0 introduces a time unit delay for neurons 31 and 35 in comparison with neurons from the main set.

It would be possible to perform the recognition based on the sets of firing neurons, as neurons from set 2 characterize digits 0 and 2 and the grouping of firing neurons into sets depends on the stimulus sequence order.

The network has a time-dependent activity. Indeed, its complex behavior is a result of time-dependent changes in network states. Each state of the network depends on its previous states. It means that if a stimulus “A” then “B” is presented to the network, “A” will produce a change in network states as a result of time-dependent dynamical behaviors and stimulus “B” will then produce a pattern of activity that represents “B” preceeded by “A”, rather than simply “B”. Therefore, the network has the ability to process time-dependent patterns.

3.3.2 Experiment 2: Sequence of five stimuli

A set of 5 patterns (digits 0-4) is used as training patterns. Each pattern was presented sequentially to the network. However, the manner of which pattern set was presented to the network is not the same as in the previous experiment. Each pattern was presented during a fixed time interval $Z$. A new pattern is presented whether the network reaches a stable state or not. In other words, we did not wait that the network reaches a stable state in order to introduce a new pattern. A new pattern is presented whenever the time interval $Z$ is over. If the stabilization time is shorter than $Z$, a new pattern is then immediately presented.

We fixed $Z$ to be equal to 200 iterations. The 5 digit patterns [0-4] were presented to the network many times. The stabilization time interval of the network when the set [0-4] has been
presented 20 times (i.e., 100 presentations of digits) to the network is reported on figure 7. For the first 25 digits presentations, the network could not reach any stable state during the time interval Z. Then, the network began to enter in a learning phase where the stabilization time is shorter than Z. After 60 presentations, the network locked into a recalling phase where it reaches stable states in a very short time (11 iterations). In this phase, the network recognized the input sequences.

As in the previous experiment, sets of synchronized neurons are observed. Figure 8 is an example of the spiking activity of the network. Based on the firing phase we can divide neurons into four sets: set 1 (neurons 2, 6, 10, 15, 18, 20, 27, 28, 30, 31, 32 and 33), set 2 (neurons 3, 7, 12, 13, 22, 23 and 24), set 3 (neurons 4, 19, 25, 26 and 34) and set 4 (neurons 8, 9, 14, 16, 21, 29 and 35). As for the previous experiment the sets of firing neurons do change depending on the stimulus sequence presentation. It should be noted that this phenomenon is observed in the nervous systems where it is suggested that the synchronization of firing activity is perhaps a means to encode information ([4]).

**Figure 7.** Stabilization time of the network when a set of 5 input patterns (digits 0-4) was presented 20 times (i.e. 100 presentations of digits).

**Figure 8.** Spiking activity of the network after being simulated sequentially by a set of five digits [0-4].

### 4 NOVELTY DETECTION

#### 4.1 Novelty Detection Based on Stabilization Time of the Spiking Network

For the proposed network model, there is no difference between training and recognition. Training is always performed except when the network is stable. As soon as signals are presented to the network, it learns by modifying the connection weights. During training, the network
oscillates. When changes on weights are too small to modify the behavior of the network, we assume that the training and the recognition have been completed. The network is then considered to be in a stable state. The time necessary to reach that state is the stabilization time (or stabilization interval). We observed from simulation experiments that the dynamical network could reach a stable state very quickly if the input signal had already been seen. Therefore, we propose a new paradigm for novelty detection by the oscillating network model.

The paradigm is comprised of two phases:

- **Learning phase**: the network with randomly initialized connection strengths is trained with learning patterns. It reaches a stable state after learning.
- **Novelty detection or recognition phase**: patterns are introduced to the trained network. The network reaches a stable state after a relatively small number of iterations if these patterns have been learned before. Otherwise, it takes a “long” time for the network to reach an equilibrium state. Based on the stabilization time, novelty detection (or recognition) can be performed by the proposed neural network model.

In the following, we present examples of this paradigm to novelty detection and recognition by the proposed neural network model.

4.1.1 **Experiment 1: Learning digit patterns [0-4]**

The learning is performed on various sequences of digit patterns without initialization of the network between presentations of patterns. In fact, living systems probably do not “initialize” their networks between stimuli.

The “clean” patterns of digits [0-4] were used to train the network. Each digit was presented sequentially to the network during 200 iterations. As in the previous section, a new
pattern was presented to the network after 200 iterations whether the network reaches a stable state or not. Also, new patterns were presented immediately whenever the network reaches a stable state before 200 iterations. In this experiment, the set of 5 patterns (0-4 digits) was presented to the network 20 times in a random order.

After completion of the learning phase, we used either the noisy versions of learned digits [0-4] or a set of “never seen” digits (5-9) to test the ability of novelty detection of the network (Pattern set in Figure 2). According to the proposed paradigm, we used the stabilization time $T$ of the network during the testing phase to decide whether a pattern has been “already seen” or “never seen” by the network. A short stabilization time means that the pattern has been already “seen”. We considered that the stabilization time is short when it is less than 100 iterations. Otherwise, it is “long”. Hereafter, we refer to the term "recognition" when the input patterns (clean or noisy) belong to the same class than the clean digits that were used for training. We refer to "novelty detection" when the input patterns (clean or noisy) do not belong to the same class than the clean digits that were used for training.

The network has a short stabilization time when testing patterns are either the clean or noisy versions of the learned digits (Table 1). In contrast, the network has a significant long stabilization time when testing digits have never been seen before. The network made recognition mistakes on 2 patterns (noisy version 1 of digit 0 and noisy version 2 of digit ) and novelty detection mistakes on 3 patterns (noisy version 1 of digit 9, noisy version 2 of digit 7 and 9). The average error percentage is 16.7% (5/30).

| Table 1. Stabilization time $T$ (in iterations) of the network trained on clean digits [0-4], tested on clean and noisy digits [0-9] |
4.1.2 Experiment 2: Learning digit patterns [5-9]

Initial training was performed on digits [5-9] and testing on digits [0-9] (Table 2). The network made more mistakes than in previous experiment: 2 recognition errors and 7 novelty detection errors. The average error percentage is 30% (9/30).

<table>
<thead>
<tr>
<th>Digits</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean version</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>595</td>
<td>371</td>
<td>595</td>
<td>371</td>
<td>371</td>
</tr>
<tr>
<td>Noisy version 1</td>
<td>199</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>283</td>
<td>583</td>
<td>667</td>
<td>371</td>
<td>11</td>
</tr>
<tr>
<td>Noisy version 2</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>283</td>
<td>11</td>
<td>667</td>
<td>583</td>
<td>11</td>
<td>667</td>
<td>11</td>
</tr>
</tbody>
</table>

4.2 Comparison with Other Networks

The experiments on noisy digits recognition show that the stabilization time of the proposed network can effectively be used as a criterion for pattern recognition. Furthermore, a performance comparison study with Hopfield, Backpropagation and DYSTAL networks helps to evaluate the proposed network for a specific recognition task (for example the novelty detection...
task in this work). Therefore, the goal of this study is not to compare these networks (including the proposed network) in a wide range of pattern recognition.

Hopfield networks are interesting from a theoretical standpoint and can be used for classification. We used the Hopfield network implemented in the MATLAB package ([8]). Multilayer networks with the backpropagation learning are most widely used as pattern recognizers in the field. Multilayer networks under the standard generalized delta rule with momentum ([9]) were used for our experiments. DYSTAL (Dynamically Stable Associative Learning) is an artificial neural network based on the features of learning and memory identified in neurobiological research on Hermissenda crassicornis and rabbit hippocampus ([2]).

By optimizing the results of each networks (regarding the same task as in section 4.1) we defined the architectures: The Hopfield network with a 7x7 matrix of 49 neurons; the multilayer network with one input layer (35 neurons), one hidden layer (5 neurons) and one output layer (5 neurons); and the DYSTAL network with an input layer (35 neurons) and an output layer with 5 neurons.

For performance comparison purposes, we used many more patterns (instead of 30 patterns as in Figure 2). For each digit [0-9], we generated a set of noisy patterns with 20% noise. Each set of noisy digit comprises 50 patterns. It means that there are 500 noisy patterns for testing. We performed the same experiments as in section 4.1. The same learning pattern sets [0-4] and [5-9] were used to train the networks. It is important to take into account the fact that each learning pattern set has only 5 clean patterns (one for each digit). Error rates (in %) with recognition and novelty detection experiments are reported for each digit in tables 3 and 4.
Table 3. Percentage of error for recognition and novelty detection: Learning on digits [0-4] and Testing on digits [0-9].

<table>
<thead>
<tr>
<th>Digits</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed network</td>
<td>52</td>
<td>34</td>
<td>26</td>
<td>26</td>
<td>40</td>
<td>0</td>
<td>28</td>
<td>14</td>
<td>28</td>
<td>20</td>
</tr>
<tr>
<td>Hopfield network</td>
<td>48</td>
<td>50</td>
<td>30</td>
<td>60</td>
<td>54</td>
<td>20</td>
<td>4</td>
<td>2</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>Backpropagation network</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>44</td>
<td>0</td>
<td>70</td>
<td>42</td>
<td>30</td>
<td>62</td>
<td>66</td>
</tr>
<tr>
<td>DYSTAL network</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>20</td>
<td>12</td>
<td>0</td>
<td>54</td>
<td>22</td>
</tr>
</tbody>
</table>

Table 4. Percentage of error for recognition and novelty detection: Learning on digits [5-9] and Testing on digits [0-9].

<table>
<thead>
<tr>
<th>Digits</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed network</td>
<td>26</td>
<td>16</td>
<td>46</td>
<td>70</td>
<td>4</td>
<td>40</td>
<td>36</td>
<td>36</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Hopfield network</td>
<td>10</td>
<td>0</td>
<td>30</td>
<td>16</td>
<td>0</td>
<td>58</td>
<td>60</td>
<td>52</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>Backpropagation network</td>
<td>50</td>
<td>36</td>
<td>58</td>
<td>54</td>
<td>42</td>
<td>24</td>
<td>6</td>
<td>8</td>
<td>34</td>
<td>6</td>
</tr>
<tr>
<td>DYSTAL network</td>
<td>2</td>
<td>0</td>
<td>28</td>
<td>38</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

A synthesized result from Tables 3 and 4 is presented in Table 5. In terms of overall error percentage, the proposed network has a performance comparable to that of the Hopfield network and better than that of the backpropagation network. However, its performance are still far from that of the DYSTAL network. It is important to note that the DYSTAL network is well adapted to systems based on correlation between patterns. Ongoing works are focused to improve the proposed network’s performance.

The Hopfield neural network is a better novelty detector in comparison with the other networks. The DYSTAL and backpropagation networks exhibit better recognition rates than
novelty detection, while the proposed network yields similar performance in recognition and novelty detection except for the novelty detection when trained on digits [0-4].

Table 5. Performance comparison in terms of percentage of error on recognition and novelty detection.

<table>
<thead>
<tr>
<th>Network</th>
<th>Learning (0-4) Testing (0-9)</th>
<th>Learning (5-9) Testing (0-9)</th>
<th>Global error percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Recognition</td>
<td>Novelty detection</td>
<td>Recognition</td>
</tr>
<tr>
<td>Proposed network</td>
<td>35.6</td>
<td>18.0</td>
<td>34.4</td>
</tr>
<tr>
<td>Hopfield network</td>
<td>48.4</td>
<td>9.6</td>
<td>53.2</td>
</tr>
<tr>
<td>Backpropagation network</td>
<td>10.4</td>
<td>54.0</td>
<td>15.6</td>
</tr>
<tr>
<td>DYSTAL network</td>
<td>1.2</td>
<td>21.6</td>
<td>3.2</td>
</tr>
</tbody>
</table>

5 DISCUSSION AND CONCLUSION

The proposed neural network model is an attempt to associate non-linear dynamical systems with neural networks. Regarding the spiking neuron model, the introduction of a variable firing threshold that depends on the firing activity of the neuron as indeed a great effect on the behavior of the proposed network.

To our knowledge, no work proposes a neuron model whose firing threshold varies depending on its own firing activity. Some authors propose a variable threshold that depends rather on the global firing activity of the network ([23]). We believe that a temporal activity at the local level (of the neuron) can yield a self-organization at the global level (of the network). Consequently, this behavior can represent spatio-temporal structure of input patterns.
We have experimentally shown that the network has characteristics that can be exploited for processing of spatio-temporal dynamic patterns. Its firing activity strongly depends on input patterns and can internally characterize the input patterns. There are several sets of neurons that fire synchronically and periodically. Each set can represent a feature of the input patterns. This grouping characteristic is very interesting as it is created by the dynamics of the network. A similar phenomenon is observed in biological nervous systems and is considered as a means to encode information.

In the framework of neural dynamics, we show by simulation that the proposed network can be considered to be a multiple attractor system. Each attractor can be associated indeed with an input pattern. Instead of characterizing the complex behavior of the network by attractors, we propose the use of stabilization time interval. In fact, convergence evaluation based on the stabilization time interval is less complicated than that based on the location of attractors. Experiments on a noisy digit recognition task shows that the stabilization time can effectively represent input patterns. We also observed that the network has a time-dependent activity. Indeed, its complex behavior is a result of time-dependent change in network states. This is a crucial criterion for the processing of time-varying patterns.

This approach does not require a supervision of the network. It is able to detect novelty or to perform recognition. In contrast to more traditional networks, learning and recognition of the proposed network are two aspects of the same dynamical process. Indeed, biological systems also seem to function in this manner where there is no separation between training and recognition.

Experiments have been presented on digits and sequences of digits in order to simplify the analysis of the internal dynamic of the network. In a future work, we will analyze the behavior of the network when non stationary stimuli are presented to the network. When working with non
stationary process (such as speech) the instantaneous grouping of neurons should be a better criteria for recognition and novelty detection than the stabilization time interval. In fact, that interval assumes the stability of the network after presentation of the stimulus. This does not seem to be realistic when working with continuous stimuli. Nevertheless, we were able to observe interesting grouping and synchronization of neurons even if the digits are stationary patterns.

In conclusion, a possible bridge between non-linear dynamics and artificial neural networks in relation with applications in pattern recognition is proposed in this paper. The paradigm that we propose is not yet mature and many work has still to be performed. But spiking neural networks with learning abilities have a strong potential and the proposed network is one of these interesting neural networks.

Acknowledgments

This work has been supported by the NSERC of Canada and by the ‘fondation’ from Université du Québec à Chicoutimi. We would like to thank Alessandro Villa (Université de Lausanne, Switzerland) for his cooperative discussions regarding this work.
REFERENCE


FIGURE LEGENDS

Figure 1. The network architecture: Samples of neighboring connections of excitatory (a) and inhibitory (b) neurons with radius equal 2 and 1 respectively.

Figure 2. Patterns of 0-9 digits (columns 1 and 4: clean digits; columns 2, 3 and 5, 6 with 20% noise).

Figure 3. Maximum weight difference \( m \) evolution when one input pattern is presented: a) digit 0 and b) digit 2. The same initial conditions are used before presenting the stimulus.

Figure 4. Spiking activity of the network after being simulated by: a) digit 0 and b) digit 2 respectively.

Figure 5. Thresholds of the network after being stimulated by: a) digit 0 only; b) digit 0 and 2 sequentially; c) digit 2 only; d) digit 2 and 0 sequentially. Each value correspondents to the threshold of each neuron after stabilization of the network.

Figure 6. Spiking activity of the network after being simulated by: a) digit 0 and 2 sequentially; b) digit 2 and 0 sequentially.

Figure 7. Stabilization time of the network when a set of 5 input patterns (digits 0-4) was presented 20 times (i.e. 100 presentations of digits).

Figure 8. Spiking activity of the network after being simulated sequentially by a set of five digits [0-4].
Figure 1. Tuong Vinh Ho, Jean Rouat
Figure 2. Tuong Vinh Ho, Jean Rouat
Figure 3. Tuong Vinh Ho, Jean Rouat
Figure 4. Tuong Vinh Ho, Jean Rouat
Figure 5. Tuong Vinh Ho, Jean Rouat
Figure 6. Tuong Vinh Ho, Jean Rouat
Figure 7. Tuong Vinh Ho, Jean Rouat
Figure 8.  Tuong Vinh Ho, Jean Rouat